Model-checking lock-sharing systems with tree automata

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Lock-sharing systems

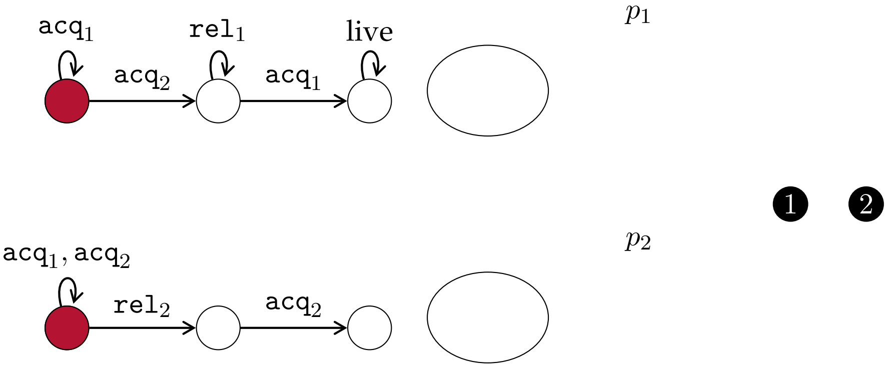
Lock-sharing system [Kahlon, Ivancic, Gupta '05]

$Proc$: set of processes
$Locks$: set of locks.

Lock-sharing system (LSS):
$\mathcal{A}_p = (S_p, \Sigma_p, \delta_p, init_p)$ for each $p \in Proc$.

Transitions include operations on locks :
$\delta_p : S_p \times \Sigma_p \rightarrow Op_T \times S_p$ with $Op_T = \{acq_t, rel_t \mid t \in T\} \cup \{nop\}$.

Semantics



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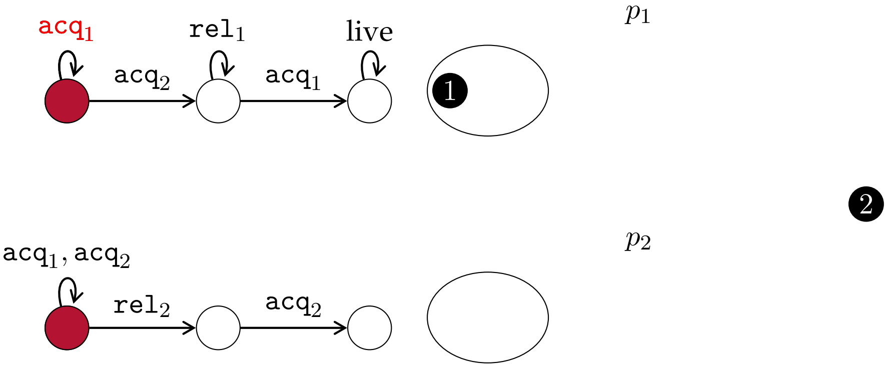

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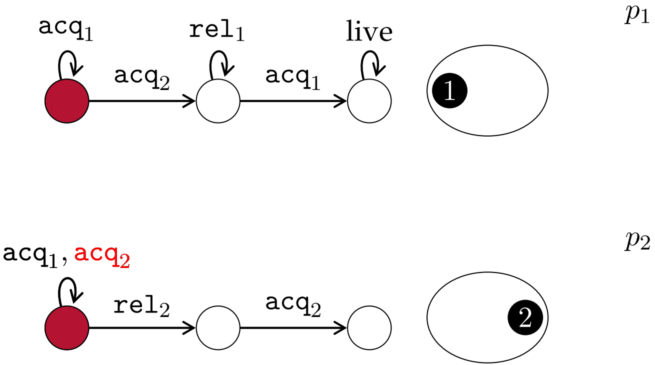

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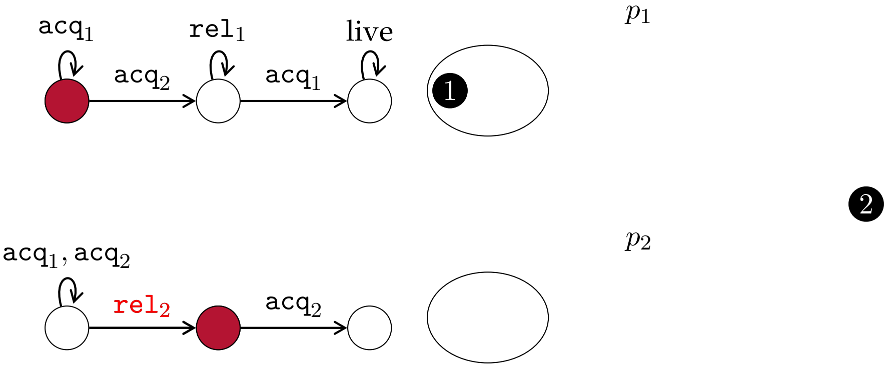

p_1
$\mathtt{acq}_1$
$\mathtt{rel}_1$
live
$\mathtt{acq}_2$
$\mathtt{acq}_1$
1
2
p_2
$\mathtt{acq}_1, \mathtt{acq}_2$
$\mathtt{rel}_2$
$\mathtt{acq}_2$

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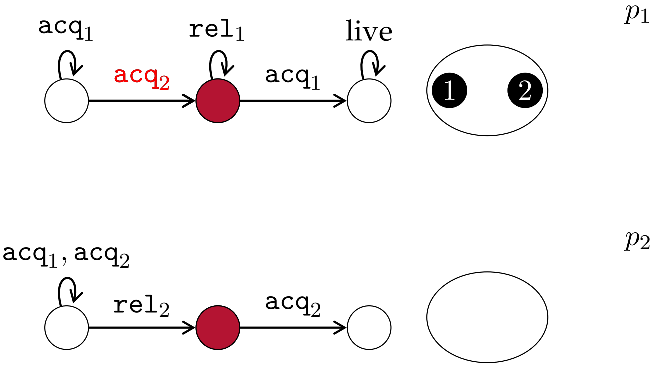

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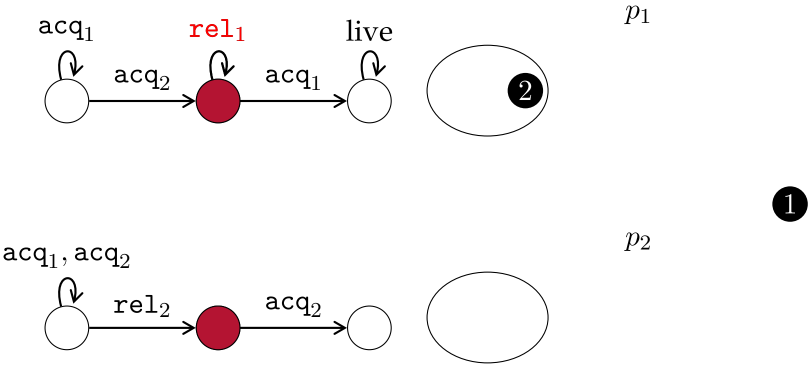

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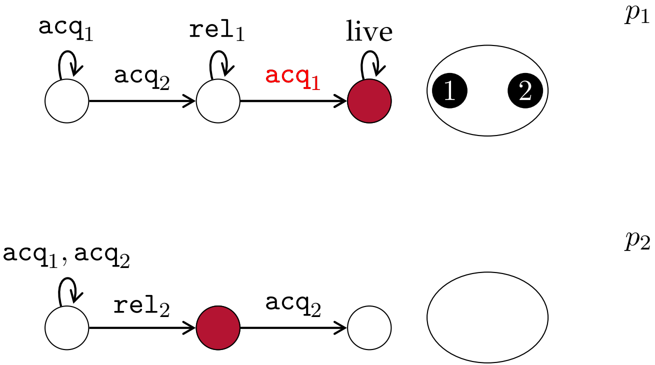

Locks vs Variables

With variables (and atomic read-write)

▷ Processes can synchronize completely.

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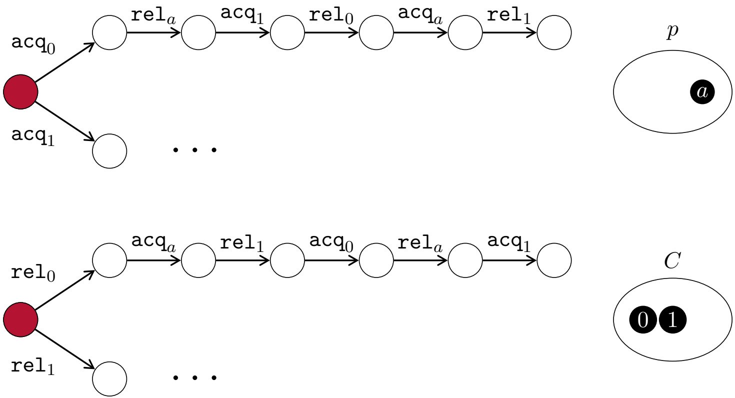
▹ No way to test if a lock is taken $\rightarrow$ Lock < Boolean variable

▹ Variables can be simulated by interleaving lock acquisitions

Passing information



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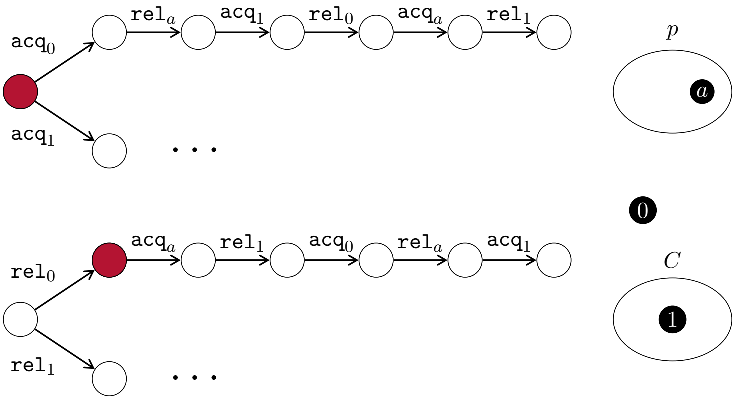

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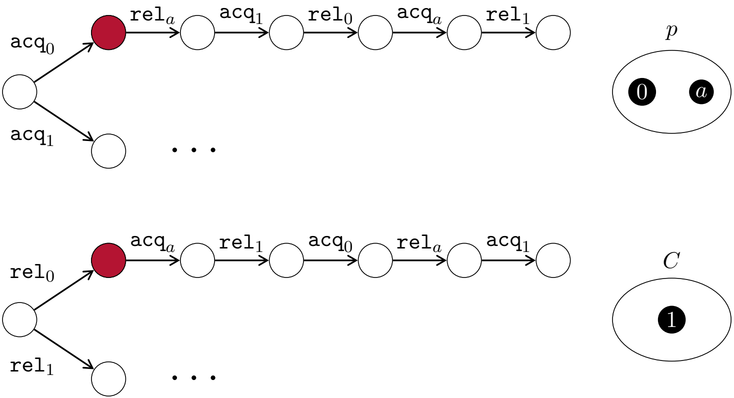

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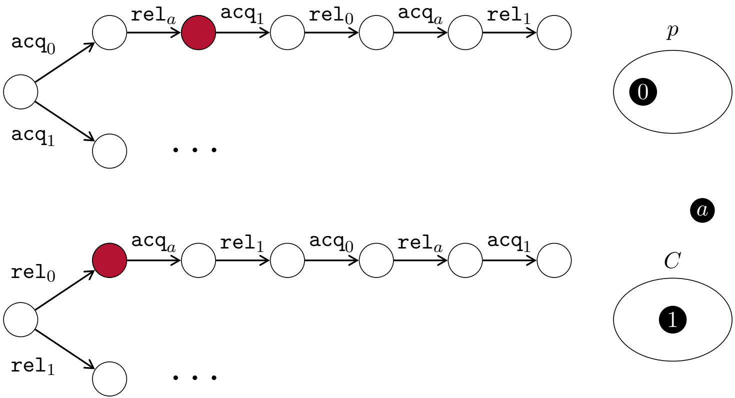

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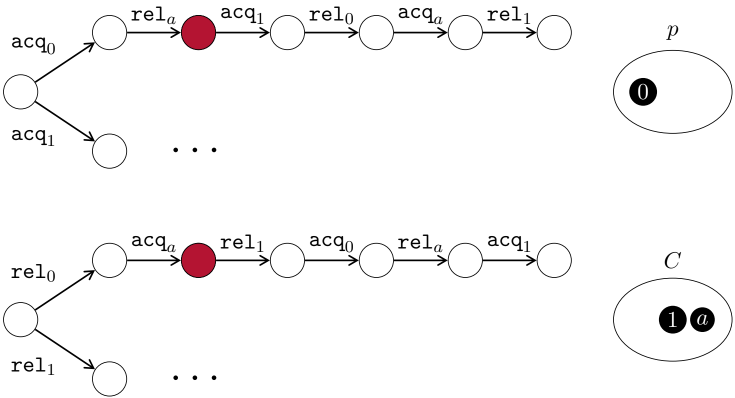

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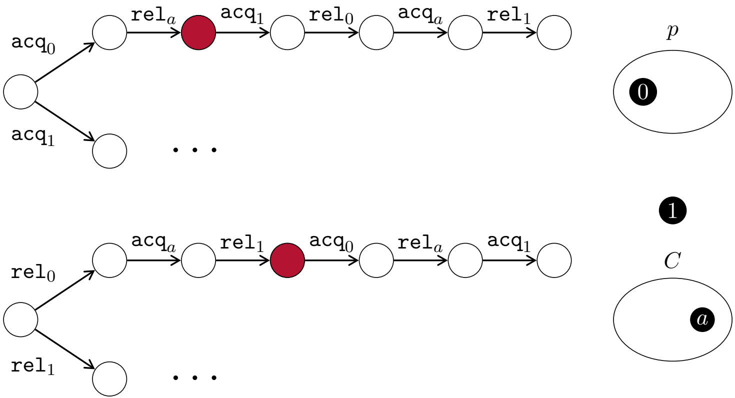

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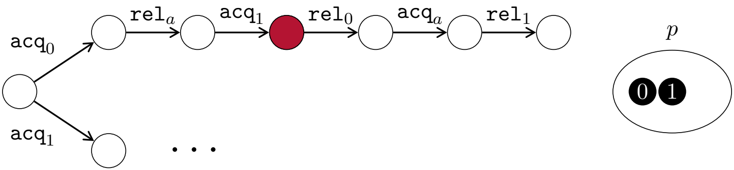

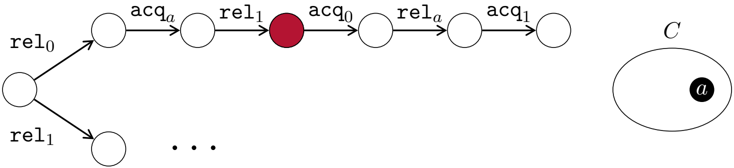

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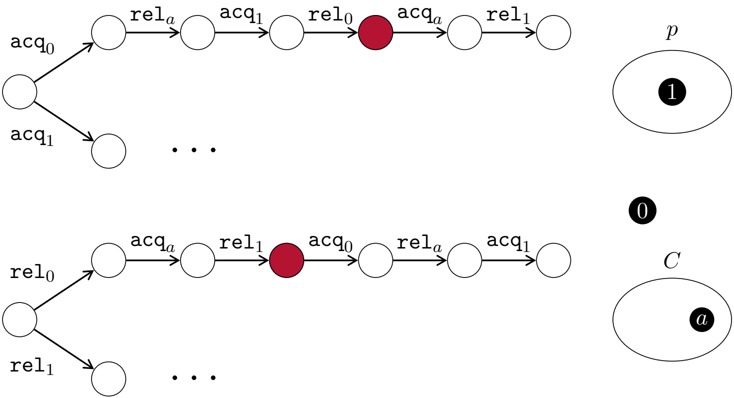

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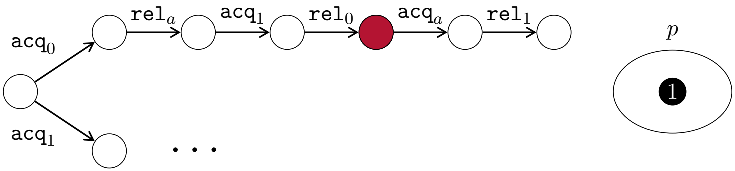

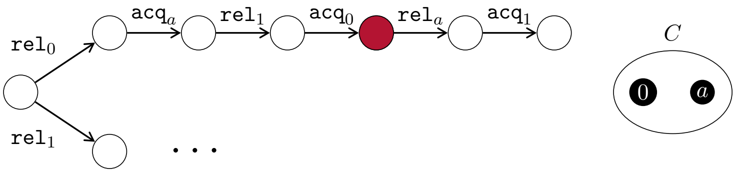

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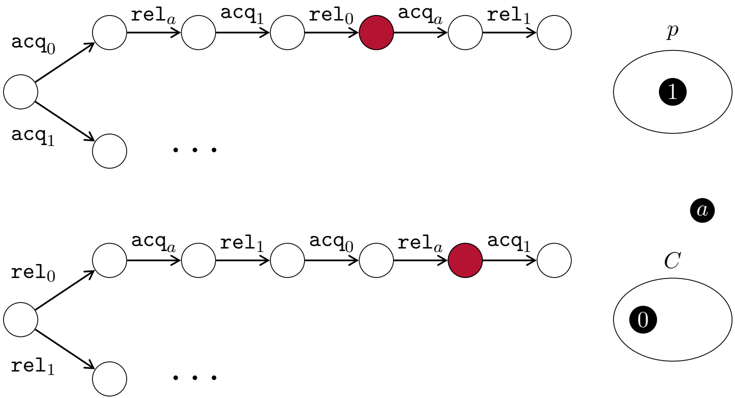

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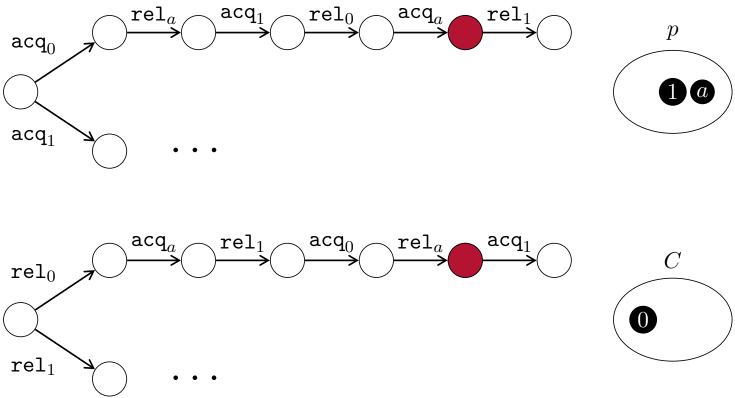

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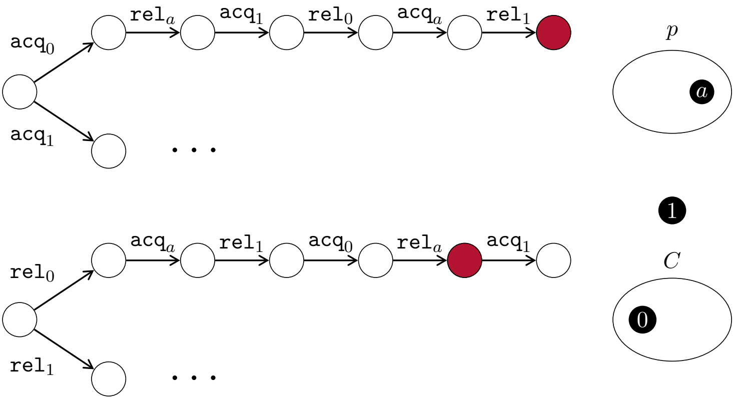

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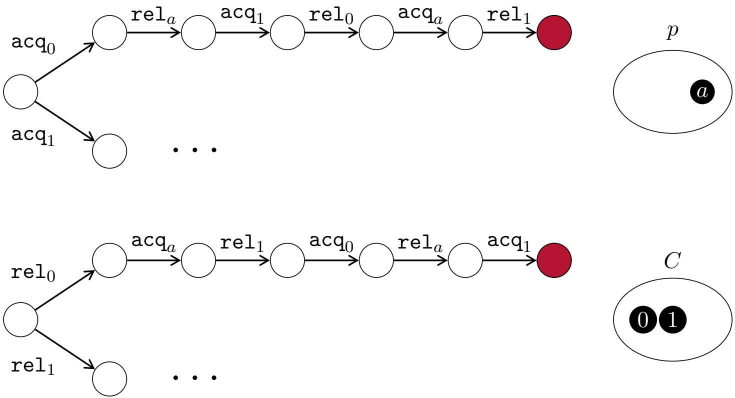
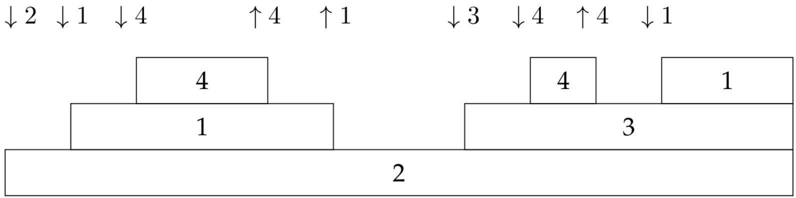


Nested locking

All processes acquire and release locks in a **stack-like order**, i.e., a process can only release the lock it acquired the latest.



Now we cannot simulate variables!

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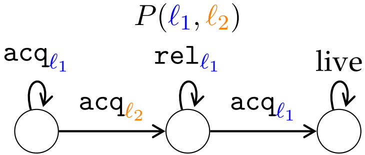
▷ A process now takes parameters, represented by *lock variables*

$$Proc = \{P(\ell_1, \ell_2), Q(\ell_1, \ell_2, \ell_3), R(), ...\}$$

Dynamic LSS

$Locks :$



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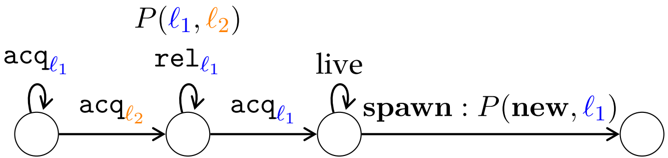

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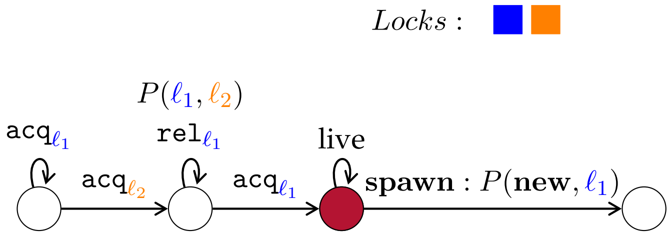

$Locks:$
$P(\ell_1, \ell_2)$
$\texttt{acq}_{\ell_1}$
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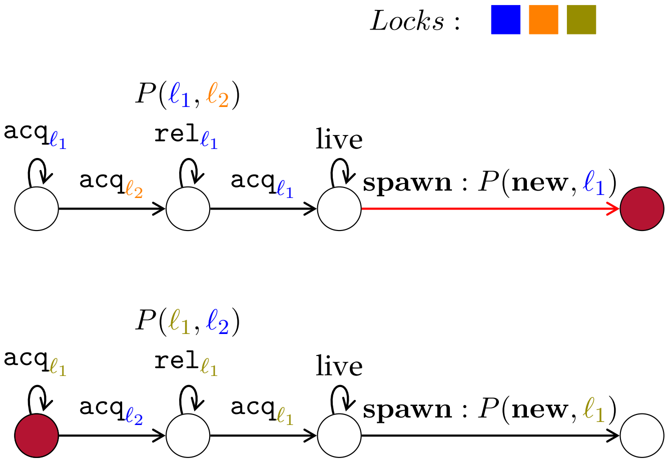
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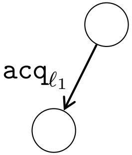

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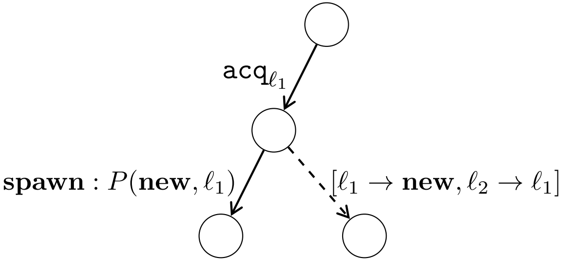

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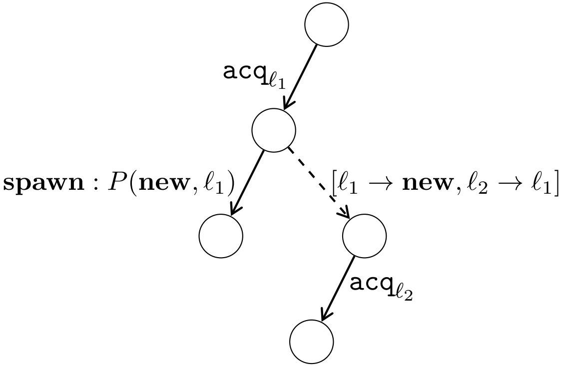

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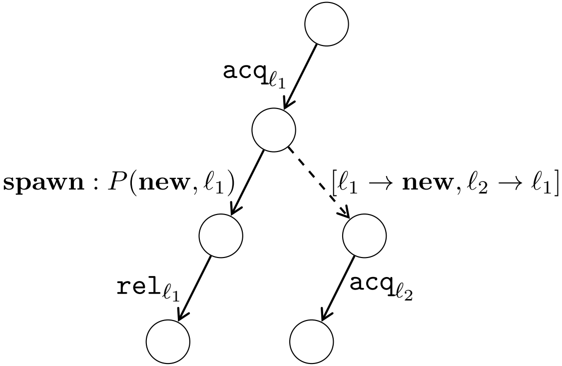

Tree specifications

We assume runs to be *fair*: If a process can execute a step infinitely many times, it eventually does.
Deadlock $\Leftrightarrow$ finite tree.

Labels

We label each node of the tree with the asymptotic behaviour of the locks associated with its variables in the subtree.

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Lemma

Consistency of those labels can be checked by an exponential Büchi automaton.

Order on locks

We also label nodes with *local orders*.
$\ell_1 \preceq \ell_2$ if ℓ_2 is taken after ℓ_1 was taken and never released.

Lemma

A tree is *schedulable* if it can be enriched with **consistent labels** and **consistent acyclic local orders**.

Result

Lemma

There exists an **exponential Büchi tree automaton** recognising realizable run trees of DLSS.
It is **polynomial** if the number of locks is fixed.

Theorem

Model-checking DLSS against regular tree specifications is EXPTIME-complete.

Right-resetting pushdown tree automata

Right-resetting = the stack is emptied every time we go to a right child.

Lemma

Emptiness is Fixed-parameter tractable for right-resetting parity pushdown tree automata.

Theorem

Model-checking pushdown DLSS against regular tree specifications is EXPTIME-complete.

Future work

Add variables ?

$\rightarrow$ Easy VASS encoding

$\rightarrow$ Find good restrictions on variables to make the problem more tractable.

Thank you for your attention!