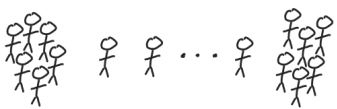
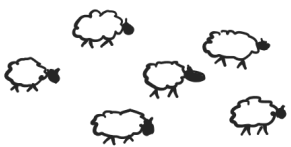
Gathering the herd
Control strategies for randomised populations



Corto Mascle

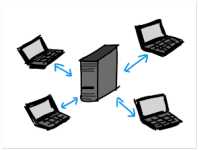
Journée EDMI 2026

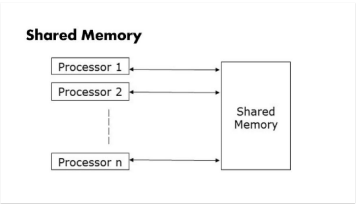



MPI-SWS
Kaiserslautern
2025 – ?
LaBRI
2021-2024


Paris
2h
2h30
MPI-SWS
Kaiserslautern
2025 – ?
LaBRI
2021-2024

Distributed systems





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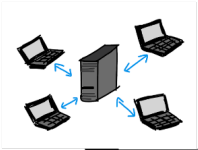
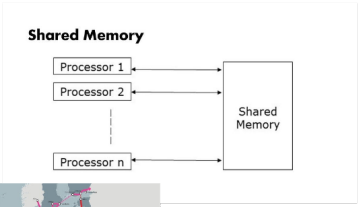
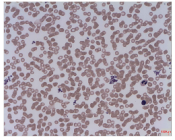
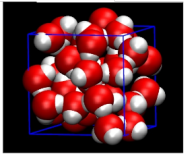
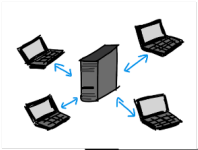


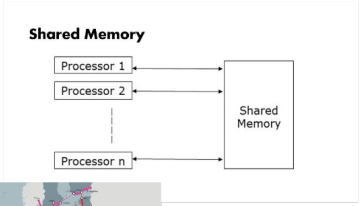


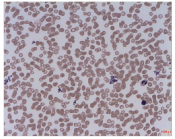


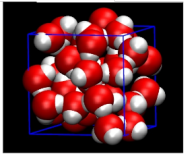


Distributed systems



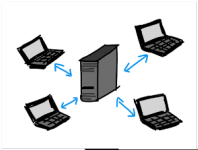


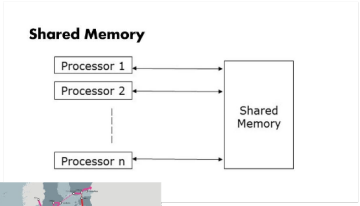


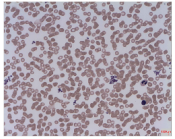


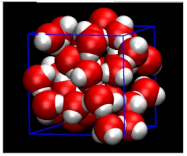
- Difficulties : combinatorial explosion, network failures, ...

Distributed systems









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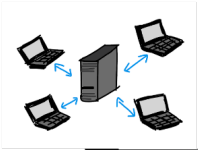

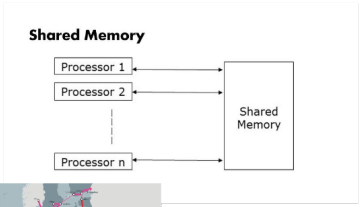

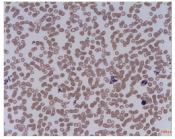

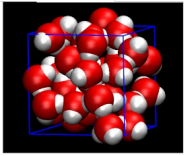

- Difficulties : combinatorial explosion, network failures, ...
- Deadlocks, starvations, race conditions, ...

Context - Models

Distributed models with an unbounded number of agents.

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Parameterized models

An arbitrary number of identical agents

Ad-hoc networks, rendez-vous protocols, ...

Dynamic models

The number of processes may increase during runtime

Multi-threaded programs, Petri nets, ...

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System >>>> **Mathematical abstraction** >>>> **Formal analysis**

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Verification

Input - System S + specification φ
Output - Does S satisfy φ

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S: Boat

φ: "

Output: Programs that maneuvers the boat so that φ holds

Control of Markovian populations

Based on work with Hugo Gimbert and Patrick Totzke

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Context

- Control a population of yeasts to make them reach a target state

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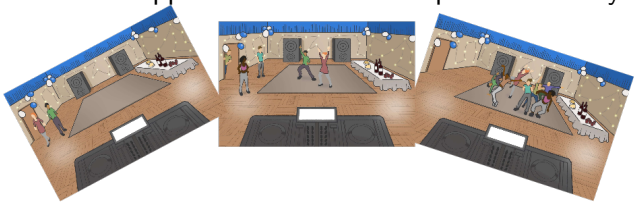
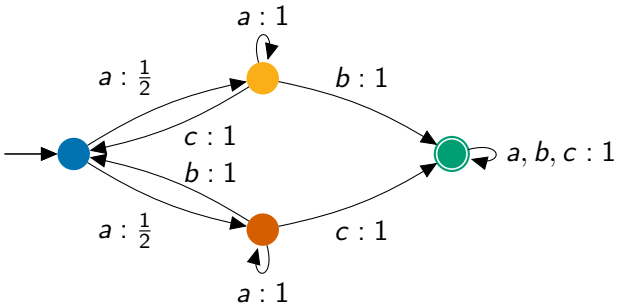
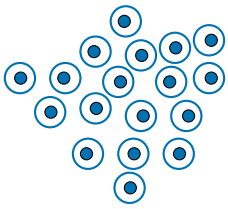


Control of Markovian processes





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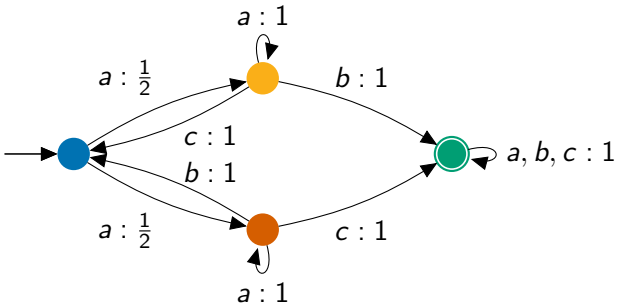

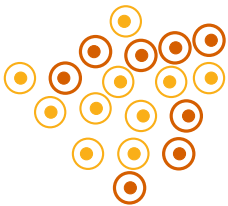

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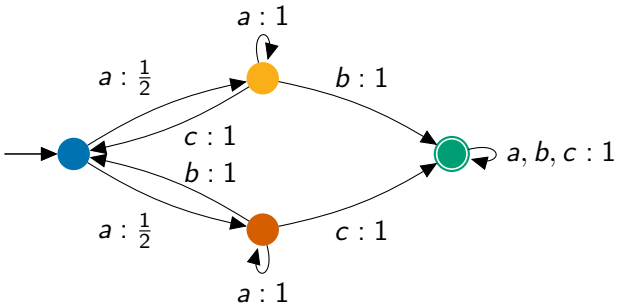

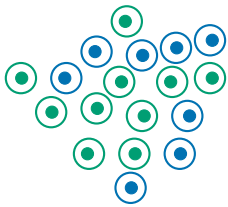

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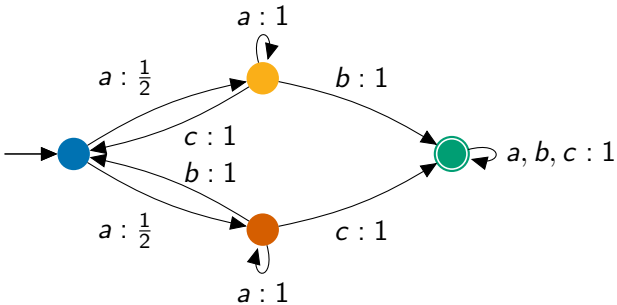

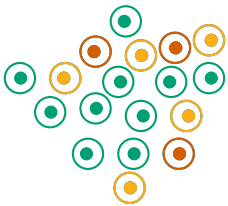

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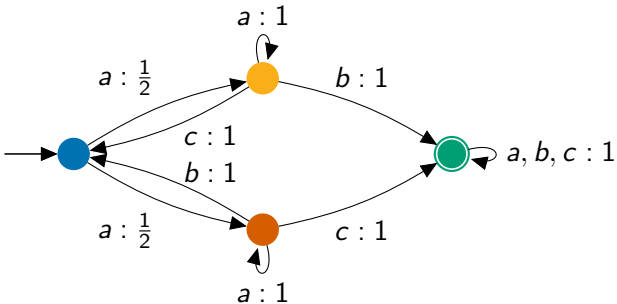

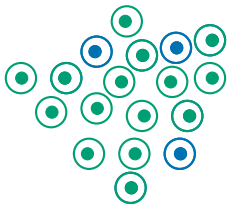

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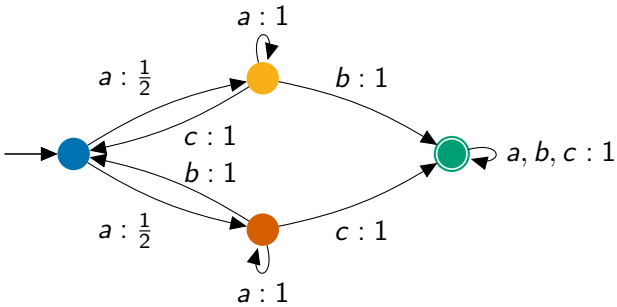

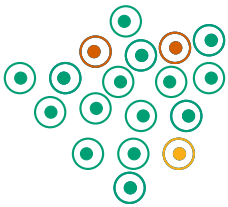

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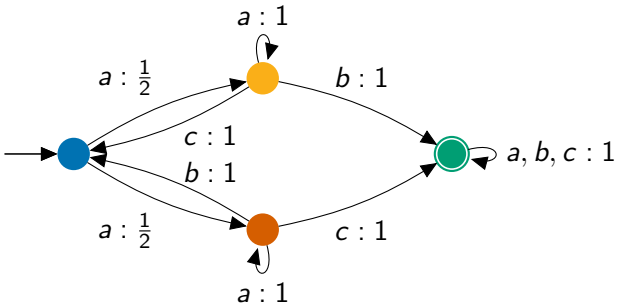

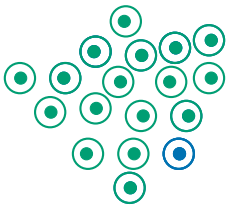

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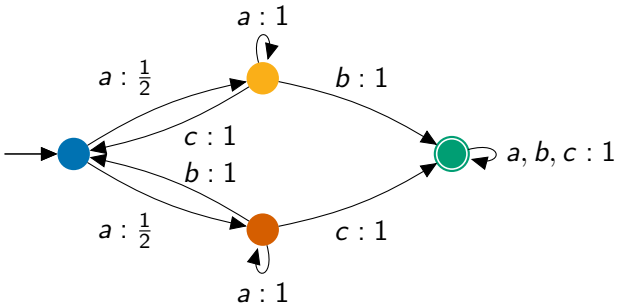

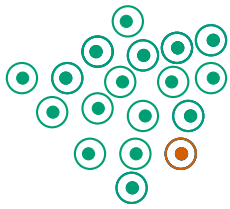

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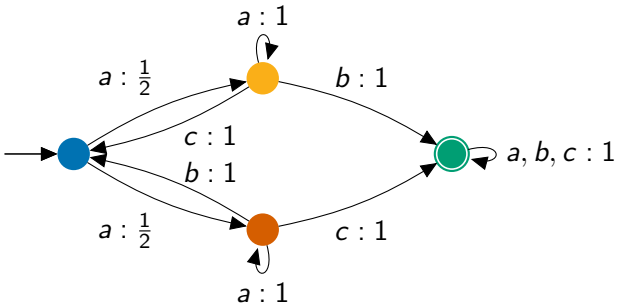

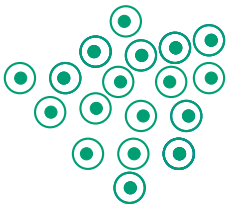

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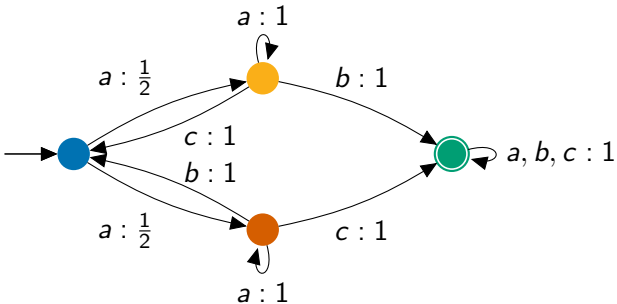
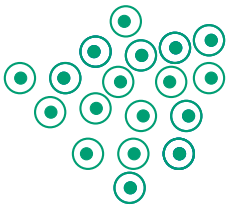
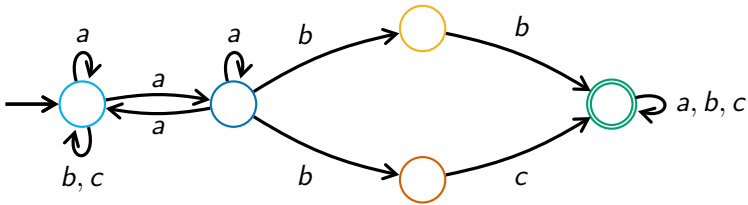




Problem: Compute a **control strategy** to **reach**  with proba 1 **for any number of cells**.

Control of Markovian processes



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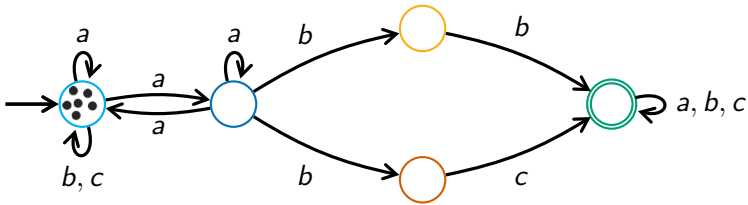

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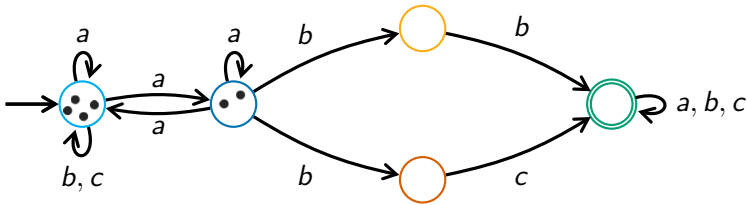

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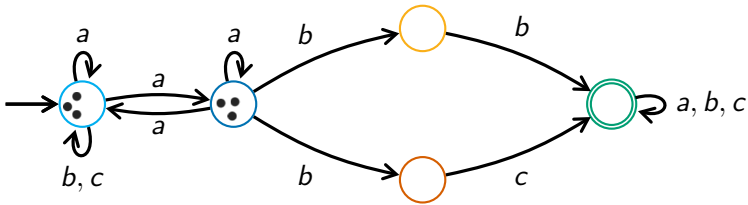

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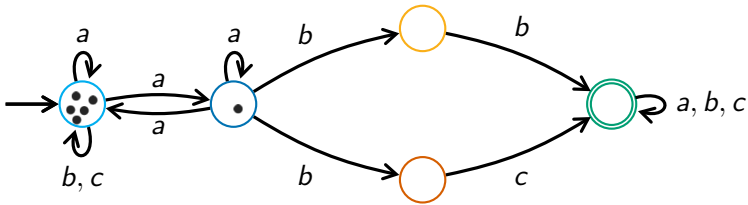

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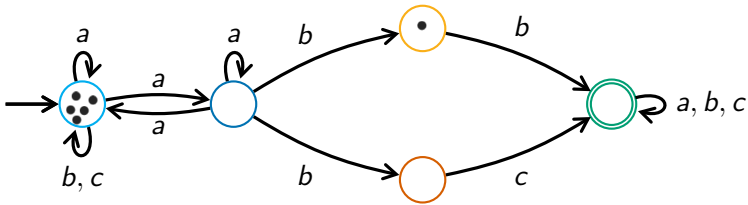

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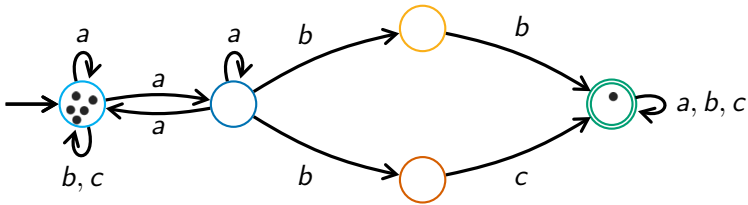

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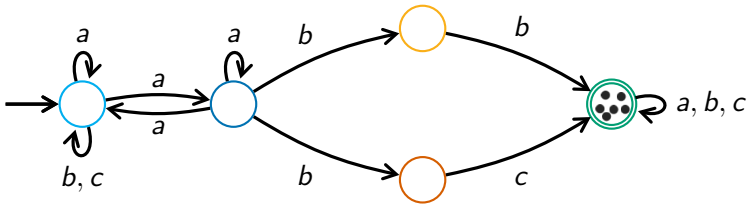
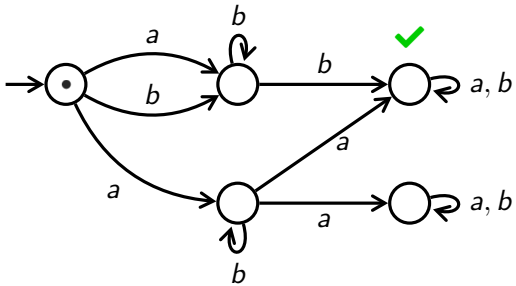


Ingredient I - Winning regions

Let's say we have only one cell.



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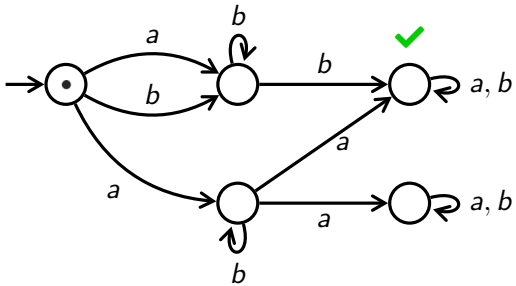

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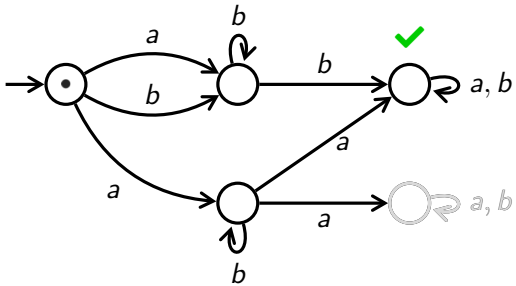

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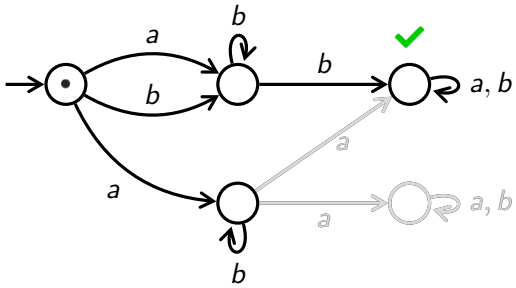

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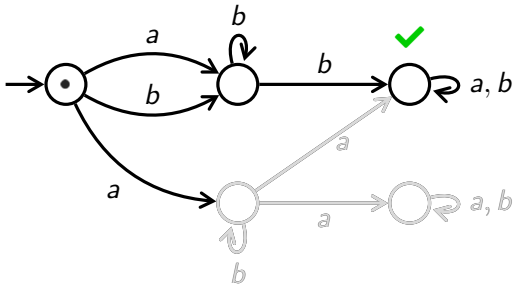

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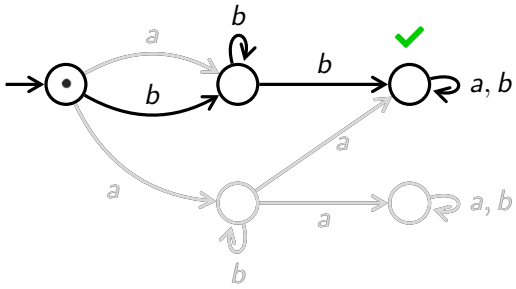

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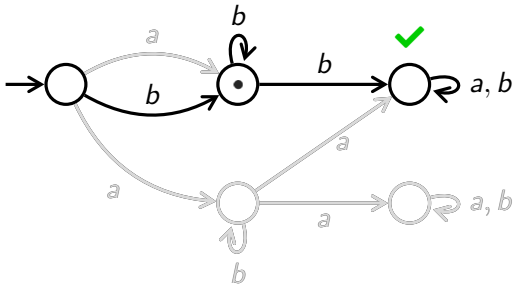

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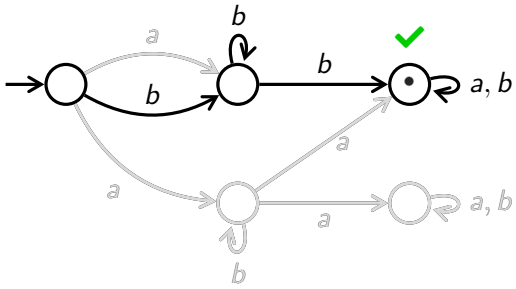

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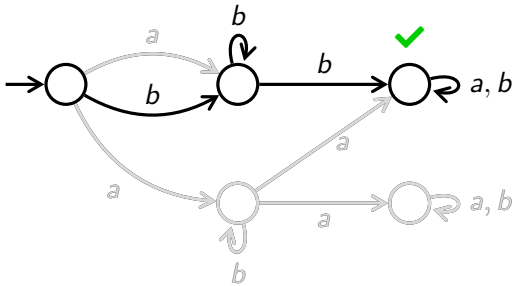

Winning region = set of configurations from which we can reach the target with probability 1.

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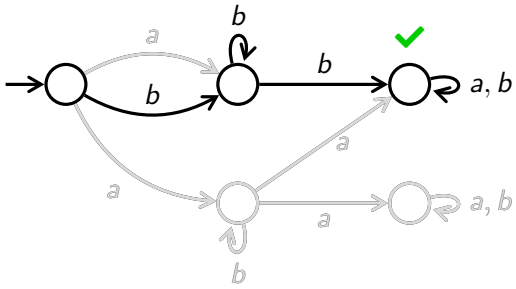
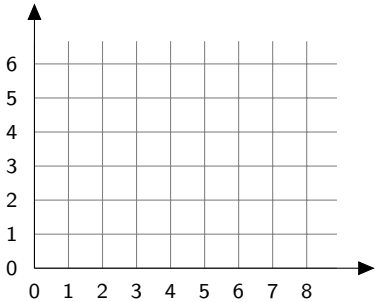


Winning region = set of configurations from which we can reach the target with probability 1.
In general, the winning region is a subset of $\mathbb{N}^S$.

Ingredient II - Well quasi-orders



- Product ordering: $(x, y) \leq (z, t)$ when $x \leq z$ and $y \leq t$.
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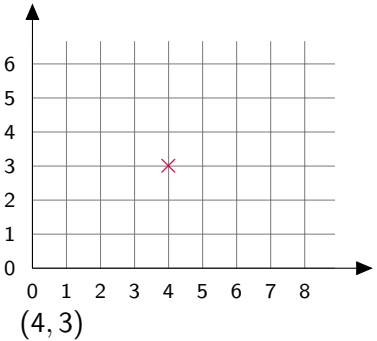

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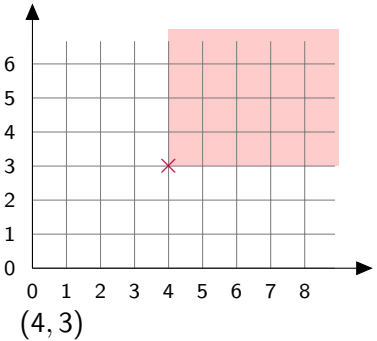

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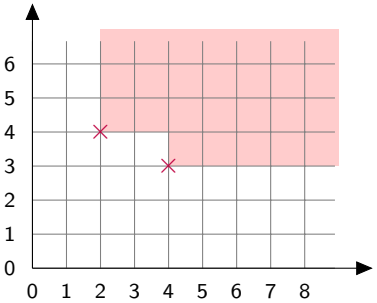

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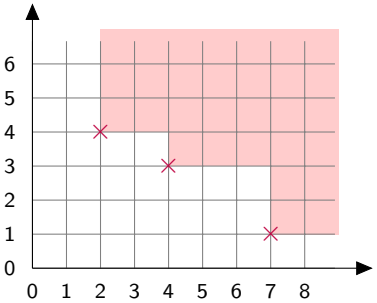

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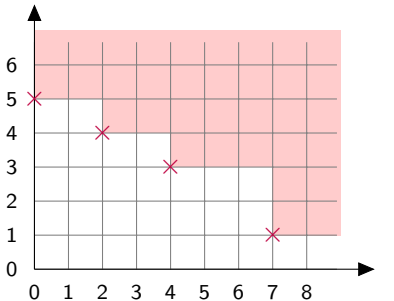

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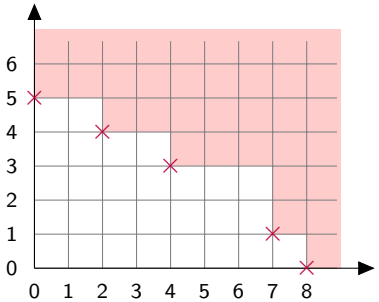

$(4,3) \to (2,4) \to (7,1) \to (0,5) \to (8,0)$

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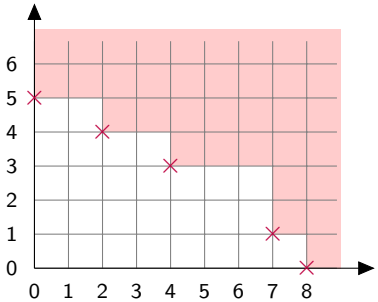

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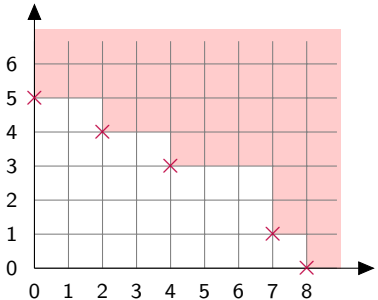

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Every set of incomparable elements for the product ordering is finite.

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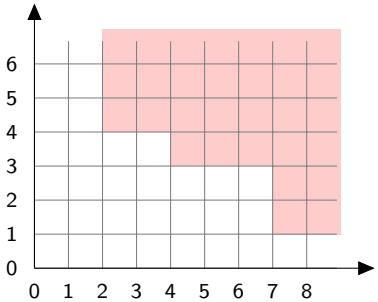

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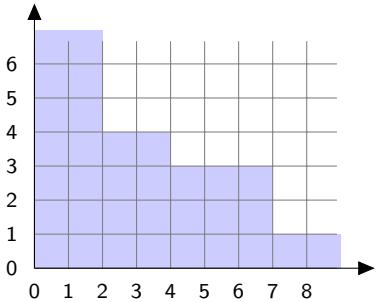

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Every set of incomparable elements for the product ordering is finite.

Every upward-closed set of $\mathbb{N}^S$ has a finite representation of the form $(2,4)\uparrow \cup (4,3)\uparrow \cup (7,1)\uparrow$.

Every downward-closed set of $\mathbb{N}^S$ has a finite representation of the form $(\omega,1)\downarrow \cup (2,\omega)\downarrow \cup (4,4)\downarrow \cup (7,3)\downarrow$.

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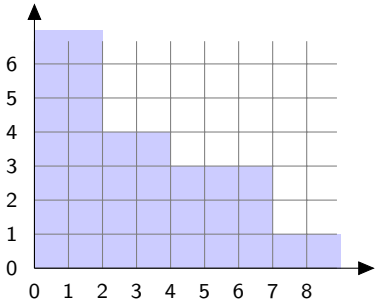

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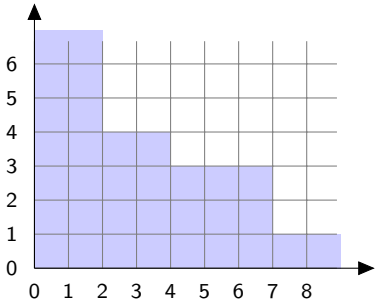

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The **winning region** of a population control instance is downward-closed.

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Set of states S
Tile = function
$S \times S \to \mathbb{N} \cup \{+\infty\}$

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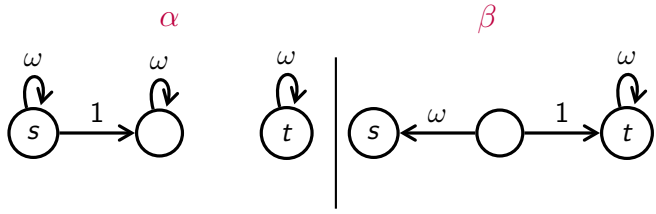

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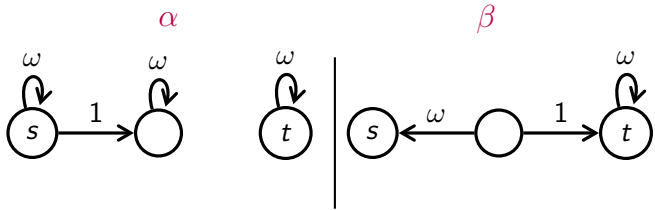

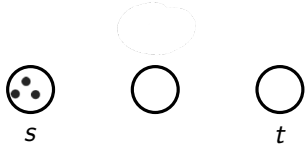

Problem [Colcombet, Fijalkow, Ohlmann '20]

Input: A set of tiles
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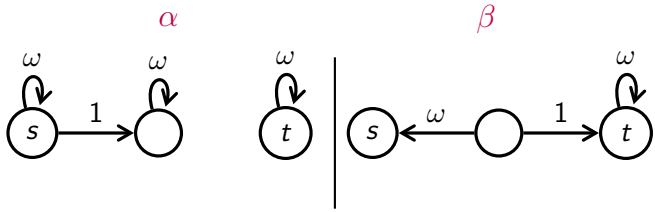

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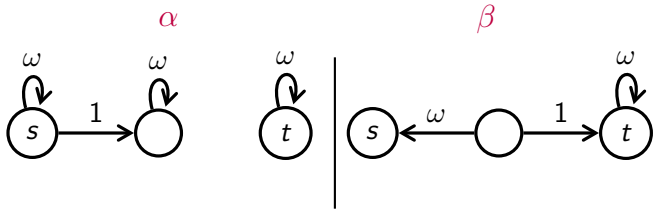

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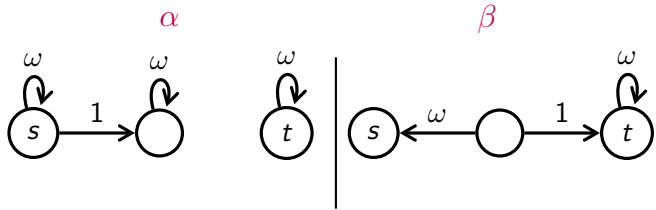

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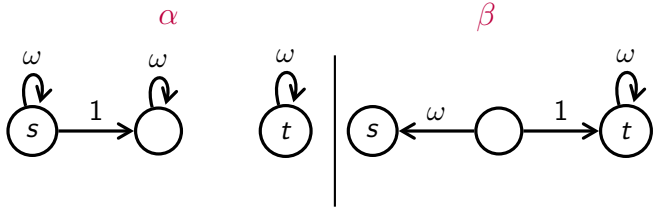

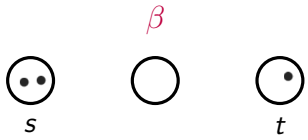

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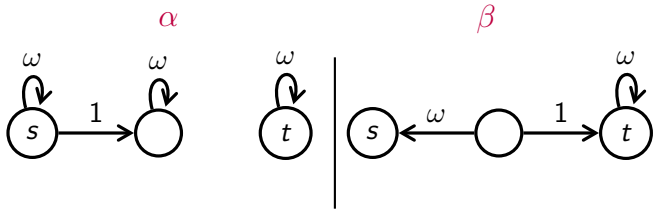

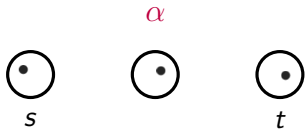

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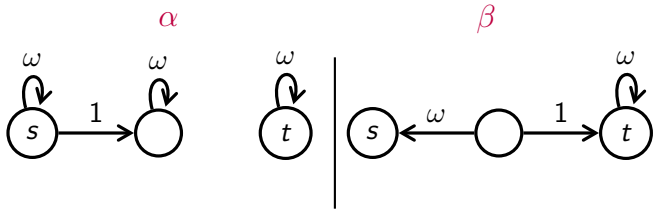

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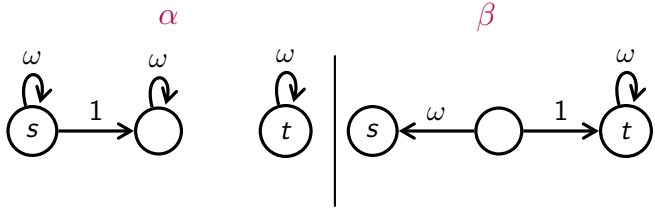

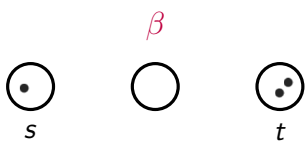

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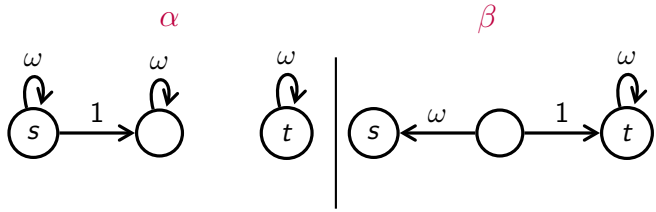

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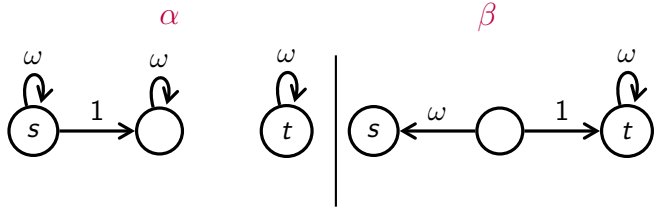

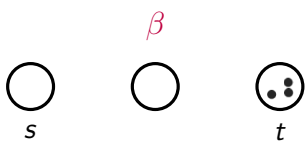

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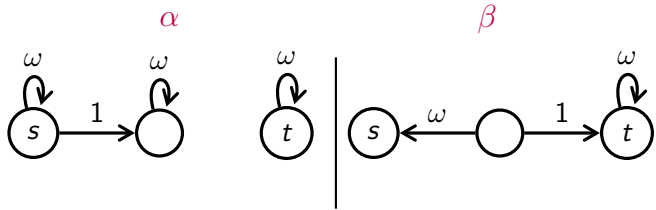

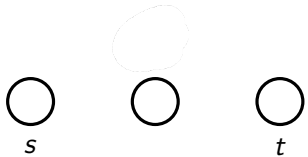

Problem [Colcombet, Fijalkow, Ohlmann '20]

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Solution: Algebra + abstractions $(\alpha^{\#}\beta)^{\#}$

Results

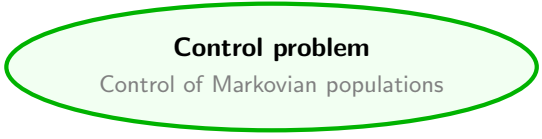
Theorem [Colcombet, Fijalkow, Ohlmann '20]

The Markovian population control problem is decidable.

Theorem [Gimbert, M., Totzke '26]

The Markovian population control problem is EXPTIME-complete.

Consequences



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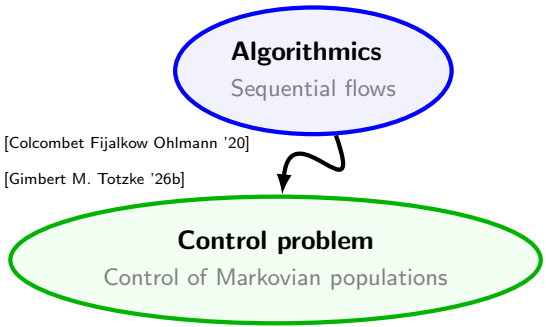

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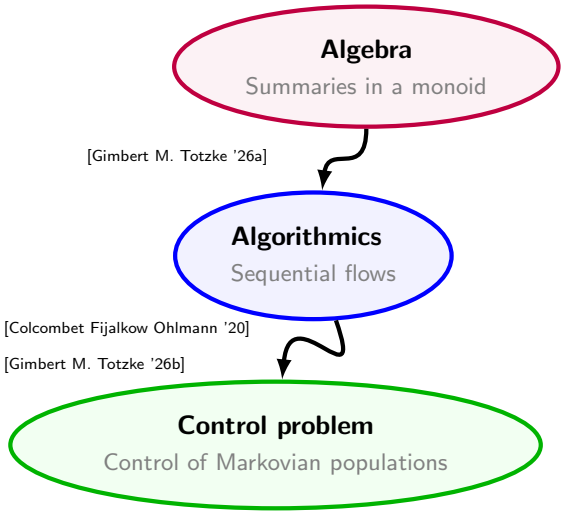

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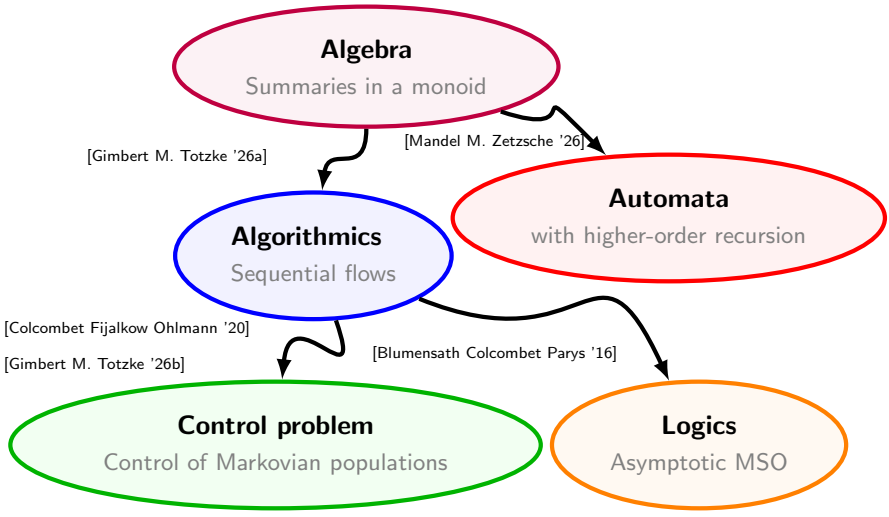

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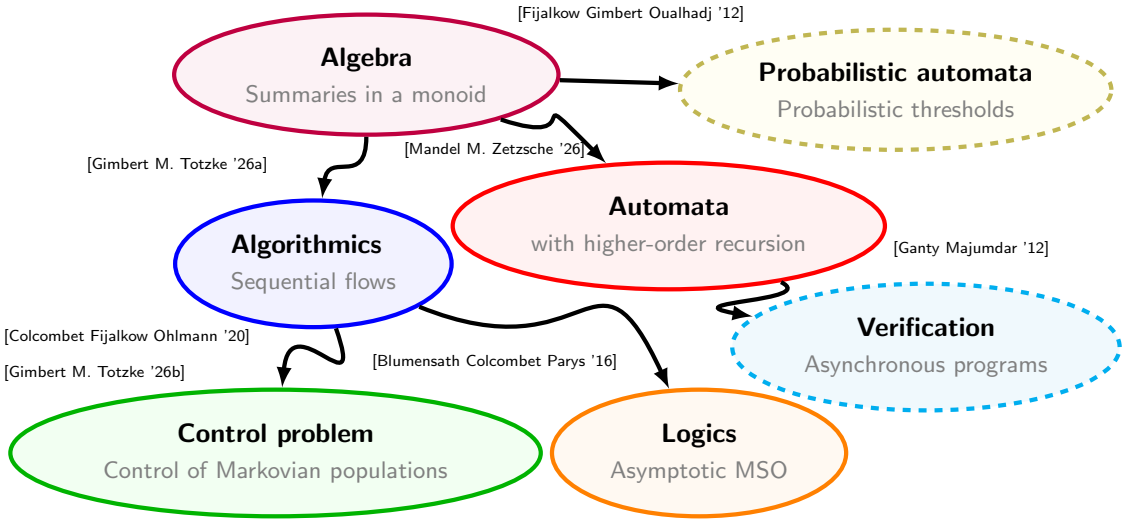

Future work

- Quantitative objectives
- Infinite-horizon specifications

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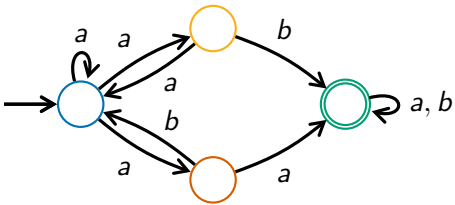
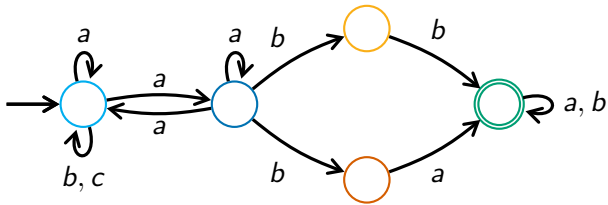
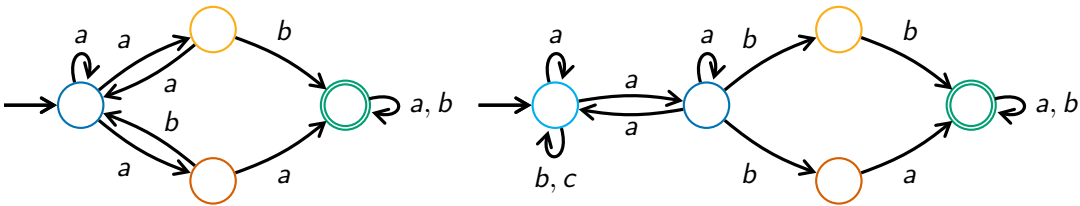
- **Fast strategies**





Future work

- Quantitative objectives
- Infinite-horizon specifications
- **Fast strategies**



Thank you for your attention!