Verification of population protocols with unordered data

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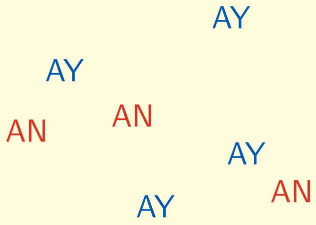

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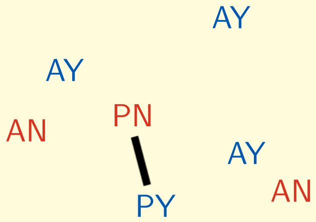

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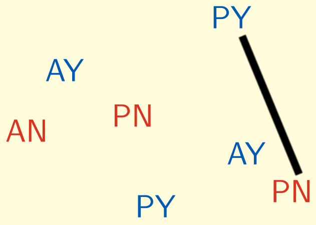

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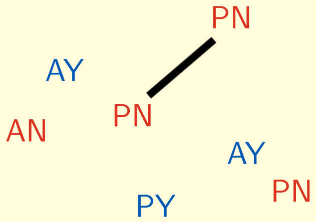

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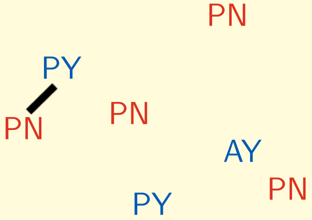

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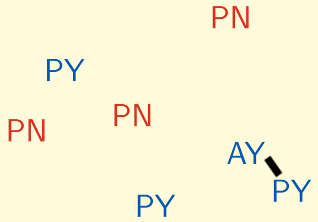

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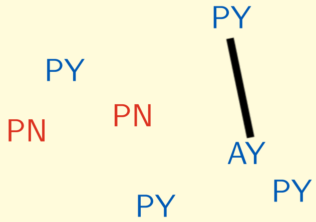

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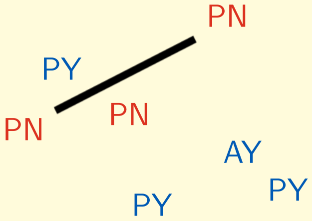

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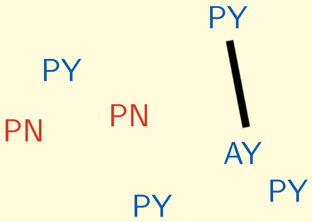

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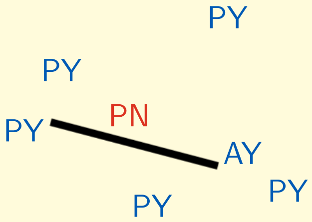

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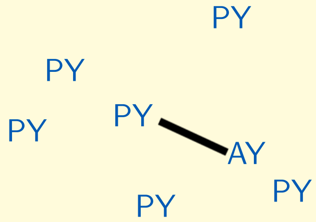

Population Protocols [Angluin, Aspnes, Diamadi, Fischer, Peralta, PODS 2004]

Finite set of states Q, with set $I \subseteq Q$ of *initial states*.
States are partitioned in two opinions $Q = Q_{Yes} \sqcup Q_{No}$
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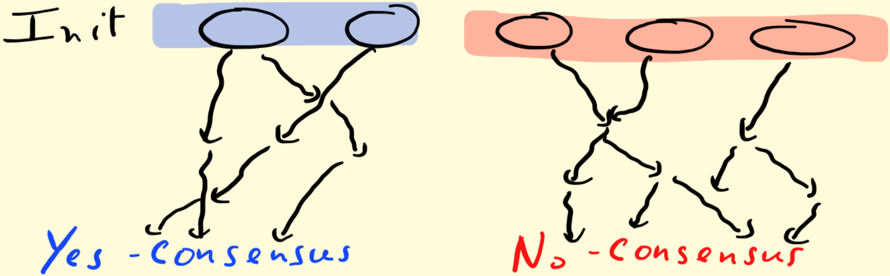

The **predicate** computed by the protocol is then the set of initial configurations from which we reach a Yes-consensus.

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Which predicates can be computed by population protocols?

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Can we check if a population protocol is well-specified? ✔

Theorem [Esparza, Ganty, Leroux, Majumdar 2017]

Checking if a population protocol is well-specified is **decidable** but as hard as Petri net reachability (Ackermann-complete).

Population Protocols with Unordered Data

Each agent carries a datum taken from an infinite set $\mathbb{D}$.
Interactions: $\Delta \subseteq Q^2 \times \{=, \neq\} \to Q^2$
Interactions take into account whether the two agents have $=$ or $\neq$ data.

$$\left.\begin{array}{l} q_0, x \\ \\ q_2, y \end{array}\right\} \xrightarrow[x \neq y]{} \left\{\begin{array}{l} q_1, x \\ \\ q_3, y \end{array}\right.$$

Majority predicate

Does some datum have more agents than all other combined?

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Open problem

What are the predicates computed by PPUD?

Well-specification problem

Given a PPUD, is it well-specified?

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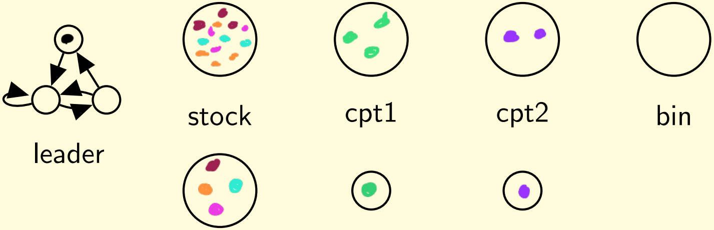
- Simulate a 2-counter machine with zero-tests.



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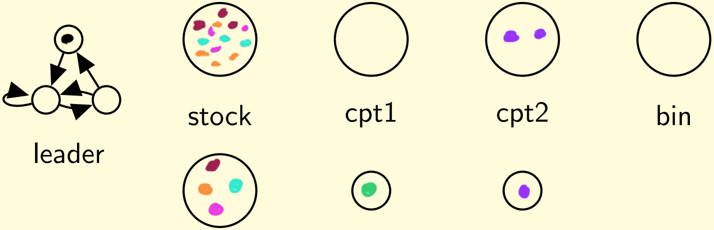

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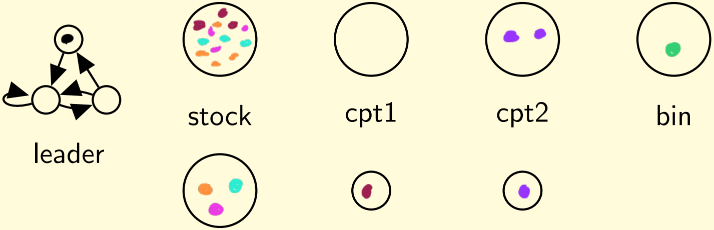

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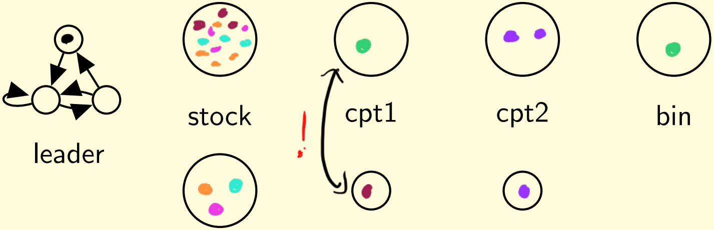

Immediate Observation

A population protocol has the **Immediate Observation** property if in every interaction one of the two agents keeps the same state.

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Theorem (Esparza, Ganty, Majumdar, Weil-Kennedy 2018)

Well-specification is PSPACE-complete for Immediate-Observation population protocols ***without data***.

Interval predicate = Boolean combination of
"At least 3 distinct data with between 1 and 3 agents in state q and 4 agents in state q'".

$$\exists d_1, d_2, d_3, \bigwedge_{i=1}^{3} (1 \leq \#(q, d_i) \leq 3) \wedge (4 \leq \#(q, d_i))$$

Theorem [Blondin, Ladouceur 2023]

The predicates computed by IOPPUD are exactly interval predicates.

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Key lemma

Given a set of configurations C described by an interval predicate, we can compute interval predicates expressing $Pre^*(C)$ and $Post^*(C)$.

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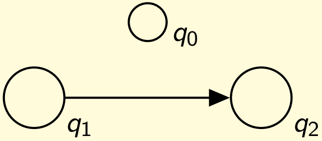
In an IOPPUD, if an agent goes from q_1 to q_2 then we can send as many agents as we want from q_1 to q_2.



IOPPUD

Generalised Reachability Expressions:

$$E ::= IP \mid E \cup E \mid E^c \mid Pre^*(E) \mid Post^*(E)$$

Theorem

Given a GRE E, we can compute an interval predicate for $[\![E]\!]_{\mathcal{P}}$.

Corollary

Given a GRE E, we can check if $[\![E]\!]_{\mathcal{P}} = \emptyset$.

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- `Well-specification`
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- `Correctness`
  = The protocol computes predicate P
- `Visible termination`
  = If at some point all agents think True (False) then they will converge to True (False)
- `Home-space problem`
  = Every fair run eventually reaches set of configurations H

Complexity

Emptiness of Generalised Reachability Expressions is:

In EXPSPACE

$\rightarrow$ By controlling the growth of coefficients when translating GRE to Interval Predicates.

NEXPTIME-hard

$\rightarrow$ By encoding the tiling of an exponential grid.

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