Verification and synthesis of dynamic systems with locks and variables

Corto Mascle
joint work with Anca Muscholl, Igor Walukiewicz

LaBRI, Bordeaux

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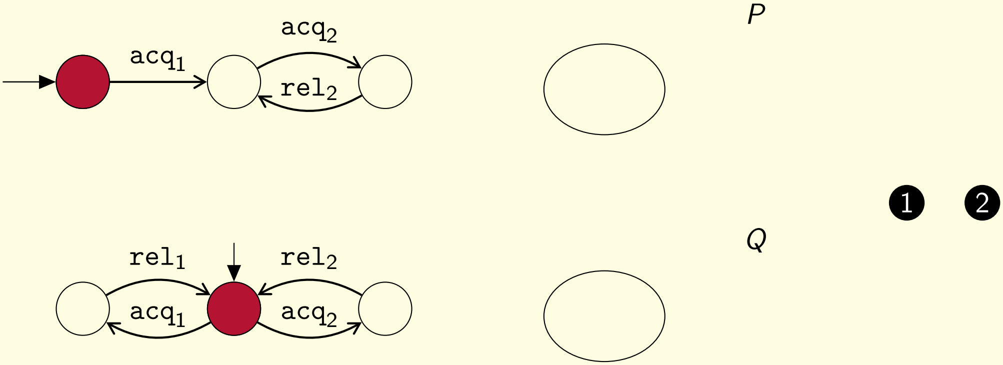
Part I

Verification

Lock-sharing systems[1]



[1]Kahlon, Ivancic, Gupta CAV 2005

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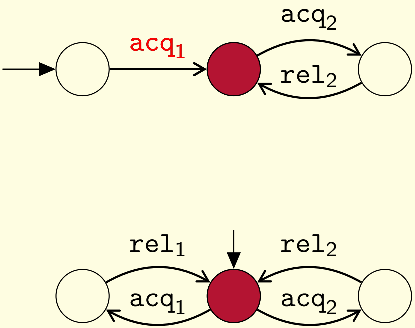

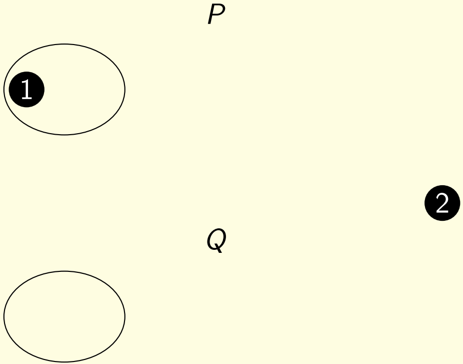

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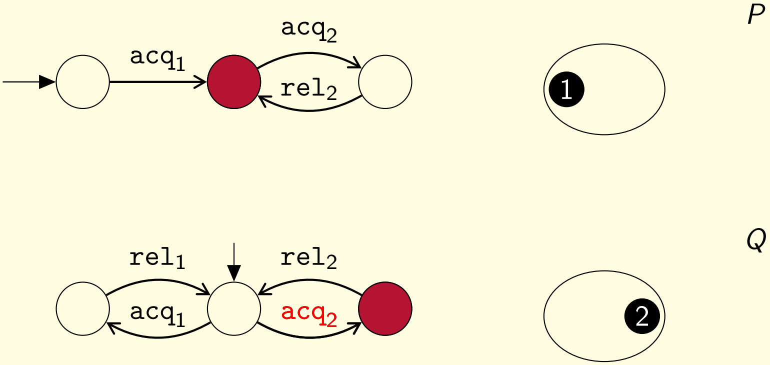

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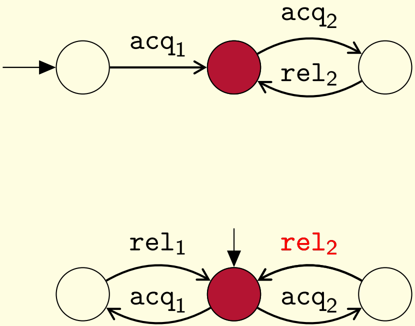

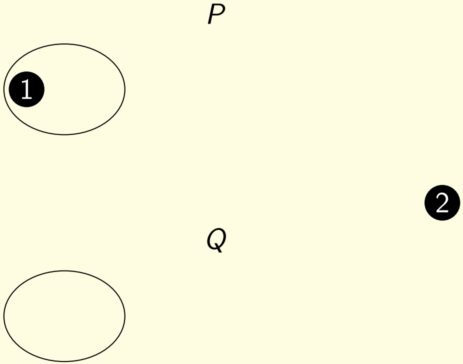

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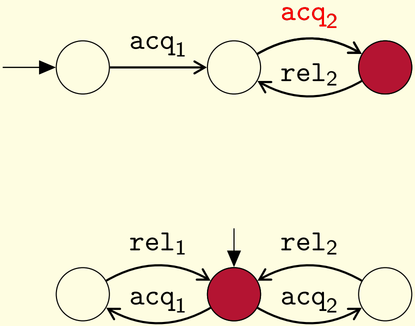

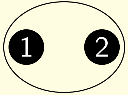

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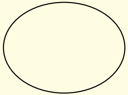

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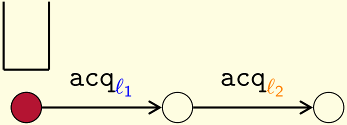
Restriction: Nested locking

All processes acquire and release locks in a **stack-like order**, i.e., a process can only release the lock it acquired the latest.



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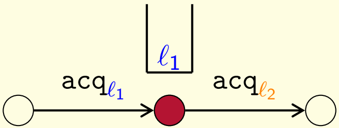

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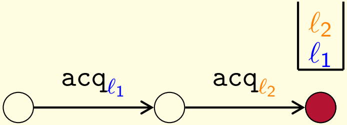

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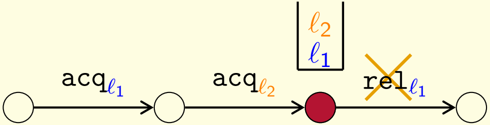

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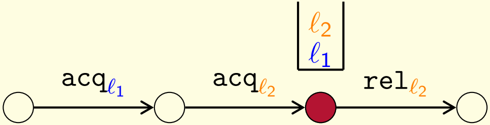

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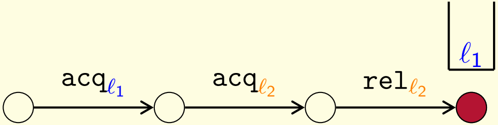

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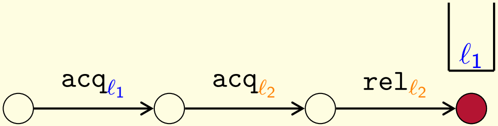

We assume nested locking in the rest of the presentation.

This restricts communication between processes.

Dynamic LSS

▷ We want to allow an unbounded number of processes and locks.

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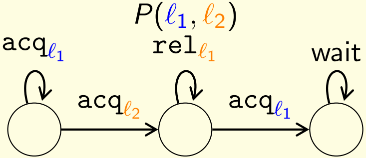
- ▷ We want to allow an unbounded number of processes and locks.
- ▷ A process can spawn other processes
- ▷ A process takes parameters, represented by *lock variables*

$$Proc = \{P(\ell_1, \ell_2), Q(\ell_1, \ell_2, \ell_3), R(), ...\}$$

Dynamic LSS[2]

Locks :



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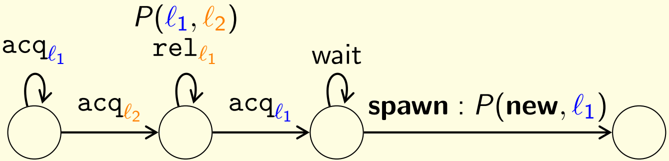

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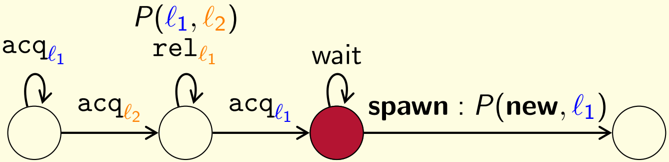

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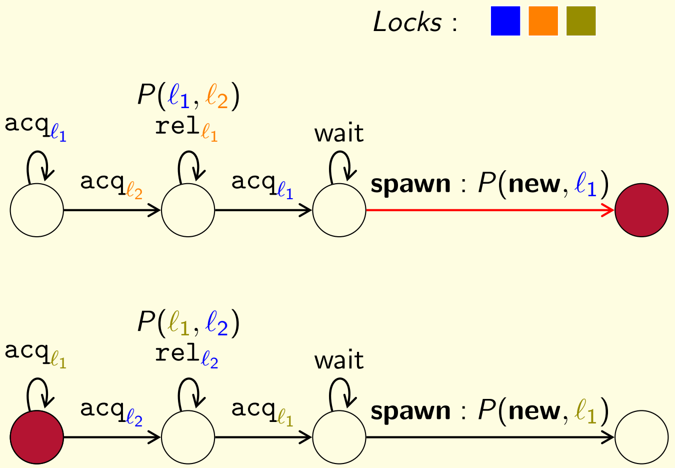

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Tree representation

$P(\ell_1, \ell_2)$

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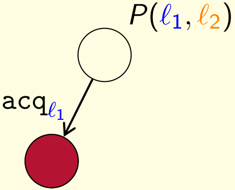

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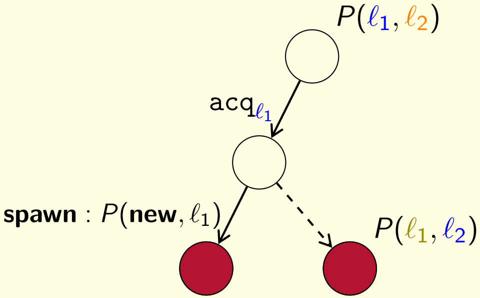

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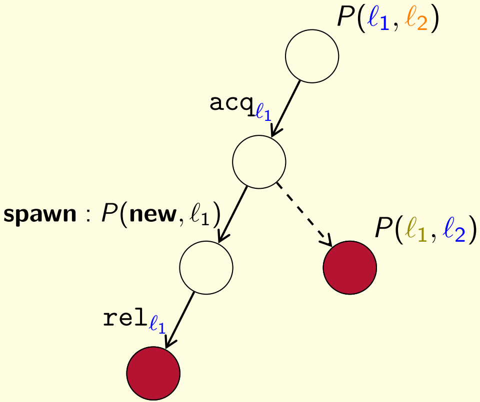

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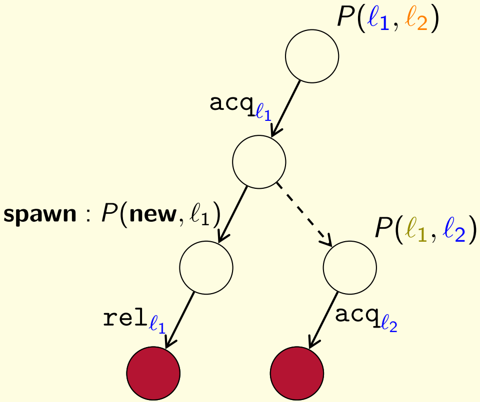

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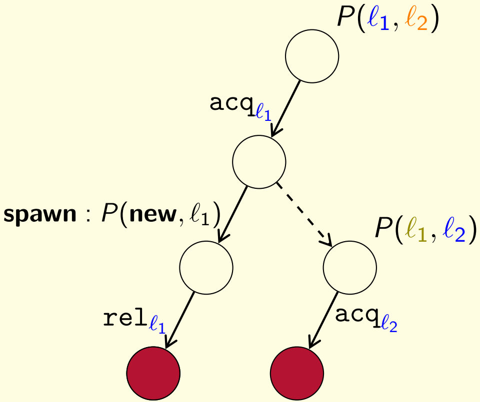
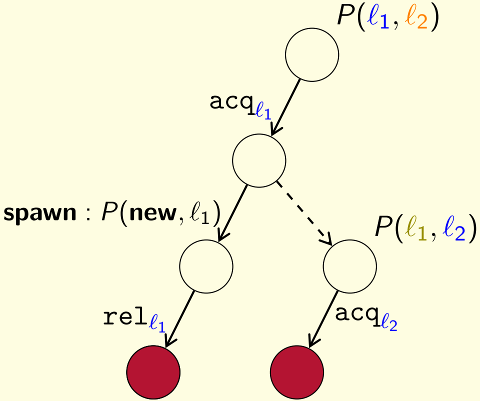


Specifications are ω-regular tree languages.

Tree representation



Specifications are ω-regular tree languages.

"Every process is blocked after some point"

"Finitely many processes are spawned"

"Infinitely many processes reach an error state q_{err}"

Deadlocks

Regular model-checking problem

Input: A DLSS $\mathcal{D}$ and a parity tree automaton $\mathcal{A}$.
Output: Is there a run of $\mathcal{D}$ accepted by $\mathcal{A}$?

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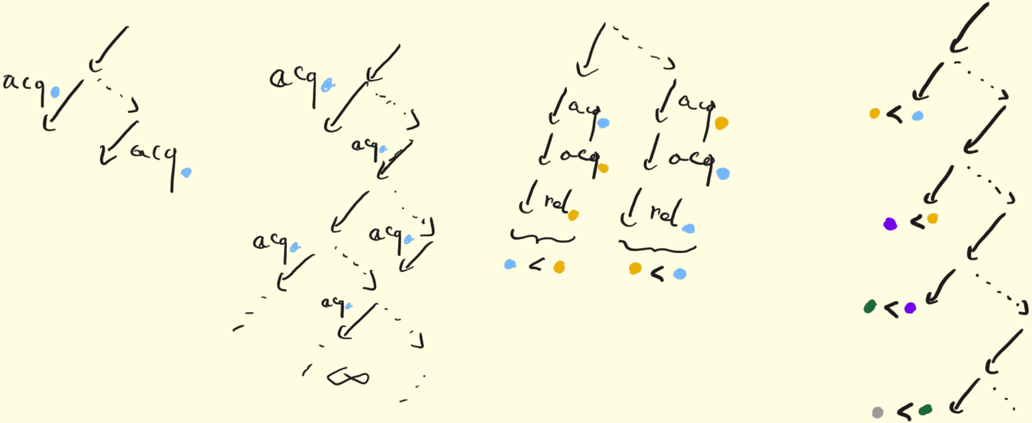
Problem: characterise trees that represent actual executions.



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$P(\ell_1, \ell_2, \ell_3, \ell_4)$

For each node we guess a label of the form

- "ℓ_1 is taken and will never be released", "ℓ_2 will be acquired infinitely many times", ...
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$$\ell_3 \begin{array}{l} < \ell_1 \\ < \ell_4 \end{array}$$

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For each node we guess a label of the form

- "ℓ_1 is taken and will never be released", "ℓ_2 will be acquired infinitely many times", ...
- $\ell_3 < \ell_1$, $\ell_3 < \ell_4$

The automaton checks that:

- the labels are consistent
- There exists a well-founded linear ordering on locks in which all local orders embed. (Technical part, also see related work [Demri Quaas, Concur '23])

Theorem [M., Muscholl, Walukiewicz Concur 2023]

Regular model-checking of DLSS is EXPTIME-complete, and
PTIME for fixed number of locks per process and parity index.

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What about pushdown processes?

Right-resetting pushdown tree automata

Right-resetting = the stack is emptied every time we go to a right child.

Lemma

Emptiness is decidable in PTIME for right-resetting parity pushdown tree automata when the parity index is fixed.

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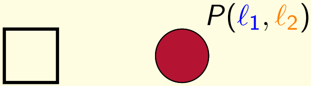
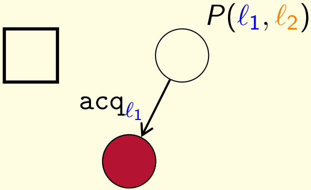
What about shared variables?

DLSS with variables



We add a register and operations `wr` and `rd` writing and reading letters from a finite alphabet in the register.

DLSS with variables



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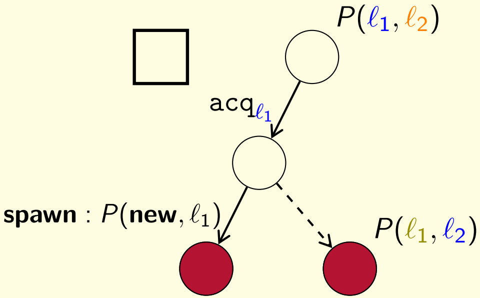

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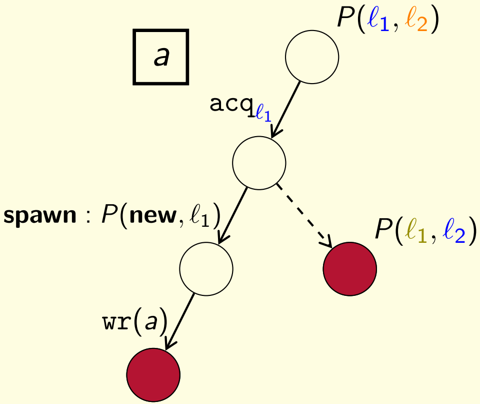

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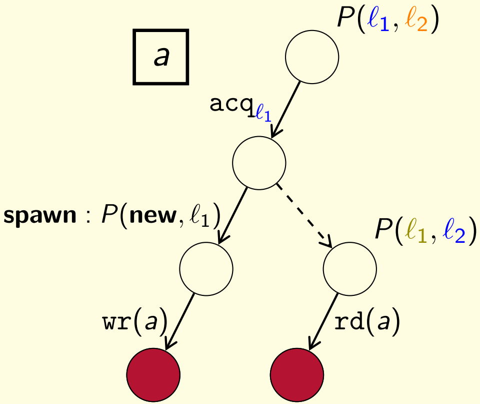

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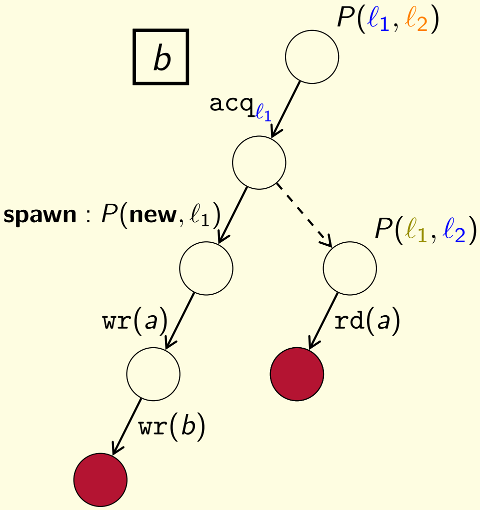

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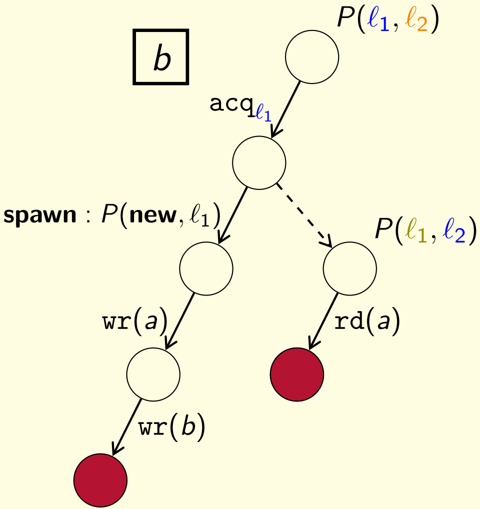

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Sets of runs are no longer regular.

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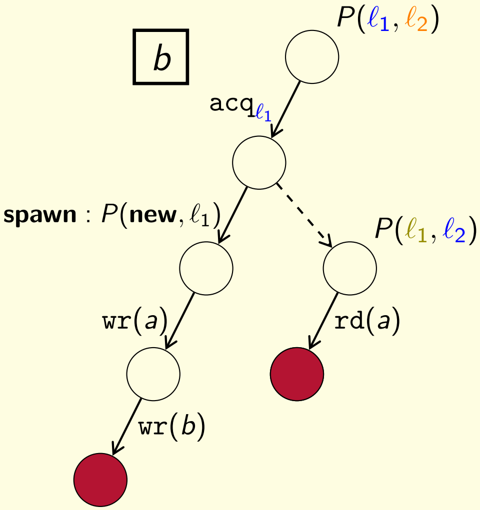

We add a register and operations `wr` and `rd` writing and reading letters from a finite alphabet in the register.

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Theorem

State reachability is undecidable for DLSS with variables.

Bounded writer reversals

Writer reversal = the process writing in the shared register changes.

[3] Atig, Bouajjani, Kumar, Saivasan FSTTCS 2014

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State reachability is decidable for DLSSV with bounded writer reversals.

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State reachability is decidable for DLSSV with bounded writer reversals.

It is undecidable when the processes are pushdown systems[3].

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Proof sketch

Consider a run with one process writing and others reading.

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Phase: run section where

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Lemma

Every finite run with a single writer can be cut into $2^{O(|Q|)}$ phases.

Proof sketch

Consider one phase.

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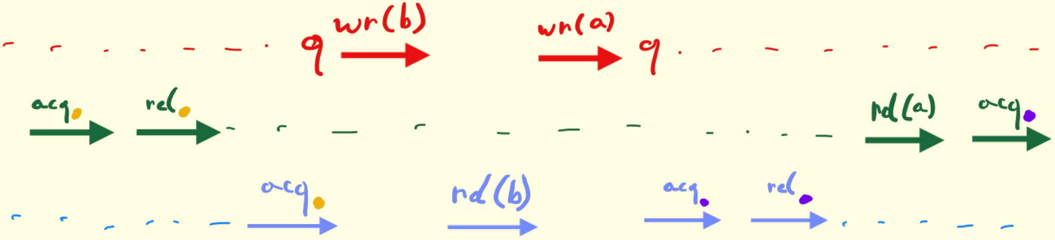
Lemma

Every phase can be replaced by a sequence of phases where at most one reader moves.



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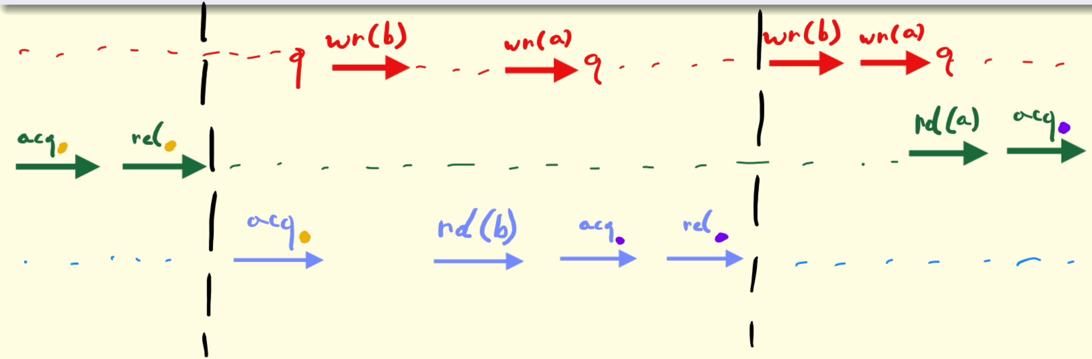

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Every phase can be replaced by a sequence of phases where at most one reader moves.

Construct $\mathcal{A}$ that:

- guesses a partition of the tree in $K2^{O(|Q|)}$ phases, each with a single writer.
- checks lock conditions
- checks compatibility of each reader with the writer

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To sum up

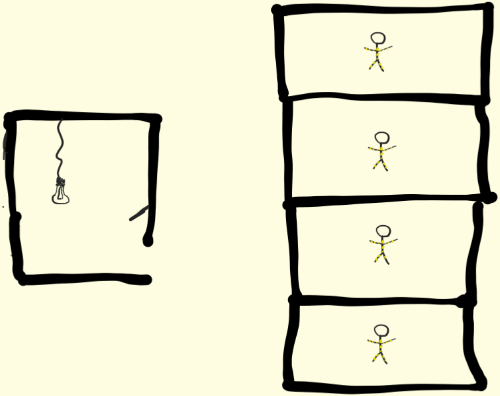
	State reach	ω-regular
Nested LSS	✓	✓
Nested Dynamic LSS	✓	✓
Nested DLSS + var with bounded v.r.	✓	?
Nested DLSS + var	✗	✗

Part II

Controller Synthesis

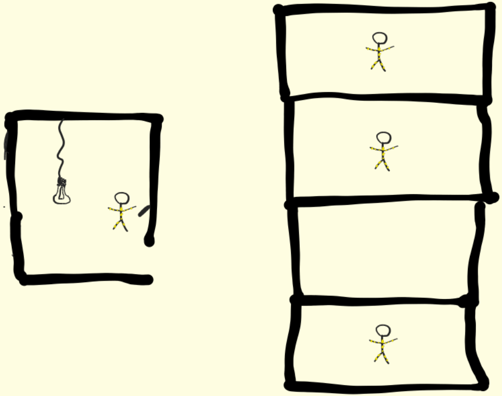
The prisoners and the lightbulb

- 4 prisoners are waiting in their cells



The prisoners and the lightbulb

- 4 prisoners are waiting in their cells
- Everyday, one is picked at random and taken to a room with a lightbulb and a switch, then brought back to the cell.



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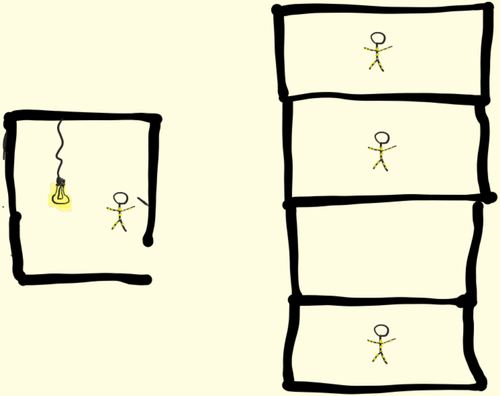

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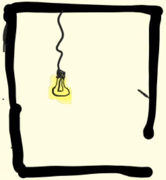

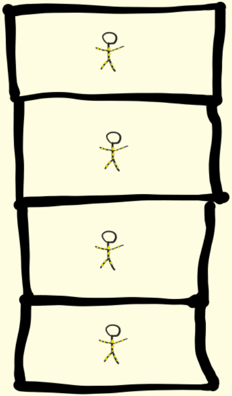

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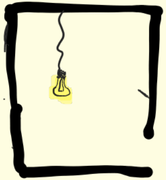
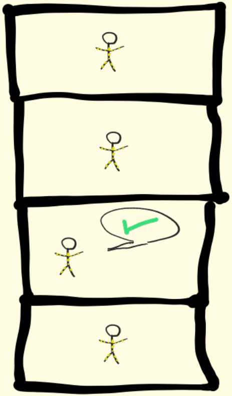




- 4 prisoners are waiting in their cells
- Everyday, one is picked at random and taken to a room with a lightbulb and a switch, then brought back to the cell.
- At any point a prisoner can claim that all prisoners have been in the cell at least once.
  They win if it is true, otherwise they lose.

Formal problem

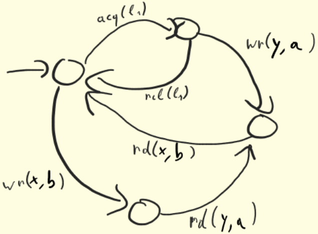
We have:

- A finite set of processes 
- A finite set of variables 
- A finite set of locks 

Each process is a finite-state transition system with operations

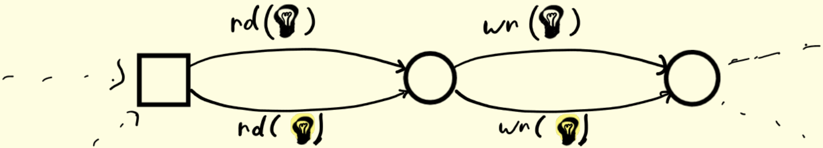
$acq(\ell)$, $rel(\ell)$

$wr(x, a)$, $rd(x, a)$



Formal problem

Processes have controllable ○ and uncontrollable □ states.



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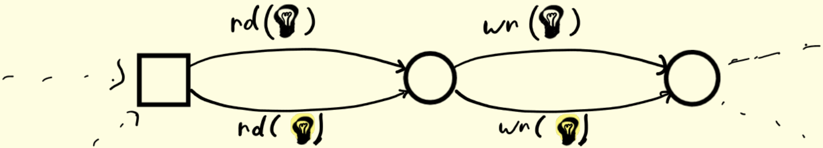
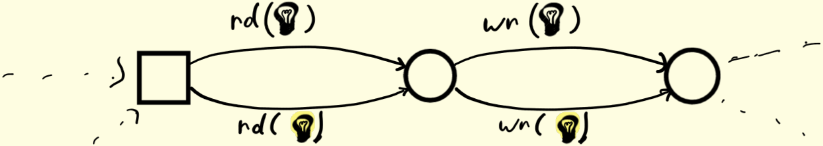


A **strategy** σ_p for process p is a function choosing the next action of p from controllable states with as input the local run seen so far.

Formal problem

Processes have controllable ○ and uncontrollable □ states.



A **strategy** σ_p for process p is a function choosing the next action of p from controllable states with as input the local run seen so far.

Specifications $=$ boolean combinations of local regular conditions.

- Processes have controllable and uncontrollable states
- Strategies are local, ie, only use the sequence of local actions of the process as input.
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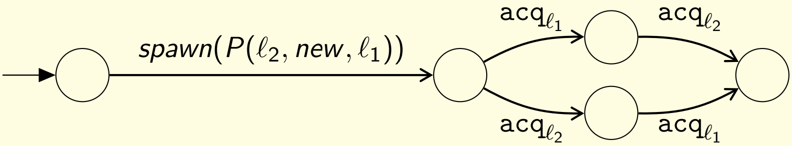
Problem

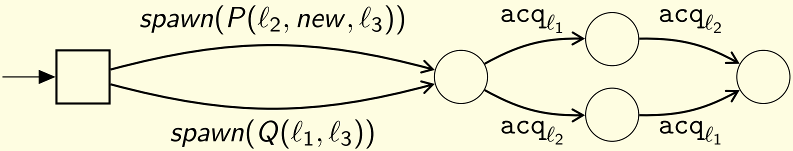
Is there a strategy $\sigma = (\sigma_p)_{p \in Proc}$ ensuring that there is no execution accepted by $\mathcal{A}$?

Example

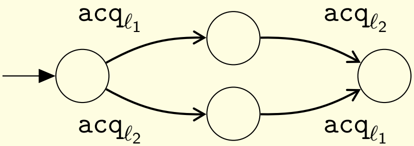
$P_{init}(\ell_1, \ell_2)$:



$P(\ell_1, \ell_2, \ell_3)$:



$Q(\ell_1, \ell_2)$:



With locks only (DLSS)

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Corollary

There is a tree automaton $\mathcal{T}$ recognising executions of $\mathcal{D}$ that are accepted by $\mathcal{A}$.

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Strategy $(\sigma_p)_{p \in Proc} \rightarrow$ set of local runs

Is there a strategy such that we cannot form a tree accepted by $\mathcal{T}$ whose left branches are those local runs?

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A strategy σ defines a set of local runs, i.e., left branches of execution trees.

A *labelled* left branch is a left branch annotated with states of $\mathcal{T}$.

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Definition

The *profile* of a labelled left branch is a tuple (p, s, π) with:

- p is the process executing in this branch
- s is the state labelling the first node of the branch
- $\pi : Prio \to 2^{Proc \times S_{\mathcal{T}}}$ is a function mapping each priority i to the set of (p', s') such that p' is spawned at a node labelled by s' while the highest priority seen before is i.

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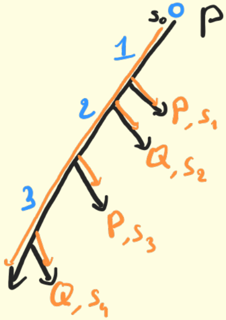

$$\rightsquigarrow \left(P,\ s_0,\ \begin{array}{l} 0 \longrightarrow \emptyset \\ 1 \longrightarrow \{(s_1, P), (s_2, Q)\} \\ 2 \longrightarrow \{(s_3, P)\} \\ 3 \longrightarrow \{(s_4, Q)\} \end{array}\right)$$

With locks only (DLSS)

The behaviour of a strategy $\sigma = (\sigma_p)_{p \in Proc}$ is the set of profiles of accepting labelled left branches (= local runs) compatible with it.

Lemma

Whether $(\sigma_p)_{p \in Proc}$ is winning only depends on its behaviour.

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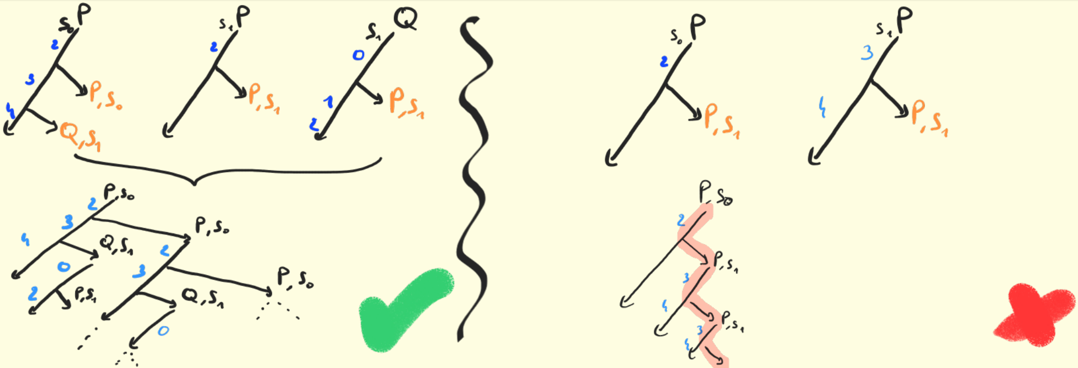

With locks only (DLSS)

Theorem

The controller synthesis problem is decidable over DLSS.

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Algorithm:

- Enumerate sets of profiles
- For each one, test whether there is a strategy yielding that set of profiles
- If there is one, $\underbrace{\text{there is one with bounded memory}}_{\text{From } \omega\text{-regular games}}$: check whether it is winning.

With locks and variables

With variables, none of this works!

- Sets of execution trees are not regular
- "Pumping argument" used for verification does not extend to adversarial setting.

What is left to do

Conjecture

Verification of nested DLSS with variables and bounded writer reversals against ω-regular tree specifications is decidable.

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Controller synthesis of nested DLSS with variables and bounded writer reversals against ω-regular tree specifications is decidable.

Formal problem

Problem

Given a system with processes, locks and variables and a specification φ, can we find a family of strategies $(\sigma_p)_{p \in \mathtt{Proc}}$ guaranteeing φ?

Problem 1

I: A system $\mathcal{S}$, $K \in \mathbb{N}$ and a specification φ
O: Is there a family of strategies $(\sigma_p)_{p \in \mathtt{Proc}}$ guaranteeing φ over runs with $\leq K$ writer reversals?

Problem 2

I: A system $\mathcal{S}$, $K \in \mathbb{N}$ and a specification φ
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Algorithm:

For $K = 0$ to $+\infty$ do
    If $\exists(\sigma_p)$ such that $\phi \wedge \leq K$ $\rightarrow$ return YES
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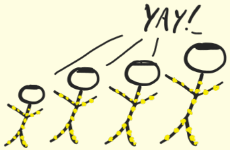

Other directions

- General approach to local controller synthesis
- Parameterised complexity
- Strategies using more information

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Thanks!