Distributed synthesis, well-quasi-orders, and memory for games

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Joint work with Anca Muscholl and Igor Walukiewicz

Highlights 2025

Asynchronous distributed systems

- Several processes running asynchronously

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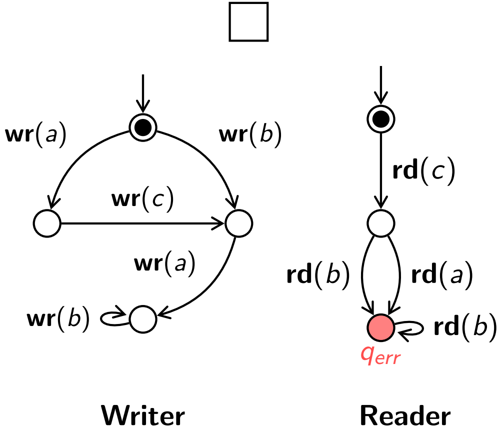

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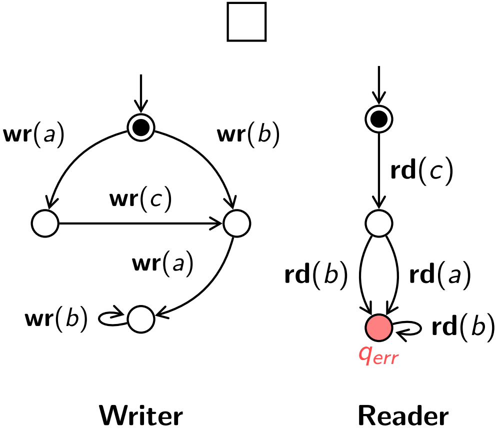

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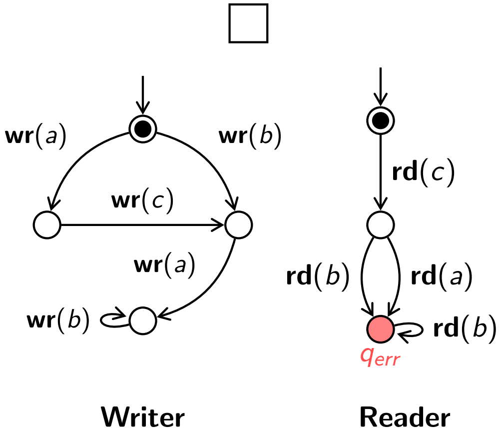

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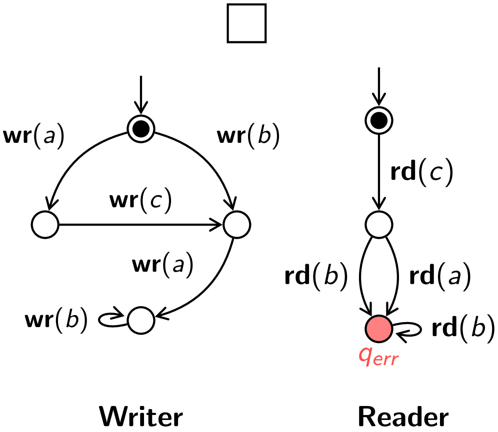

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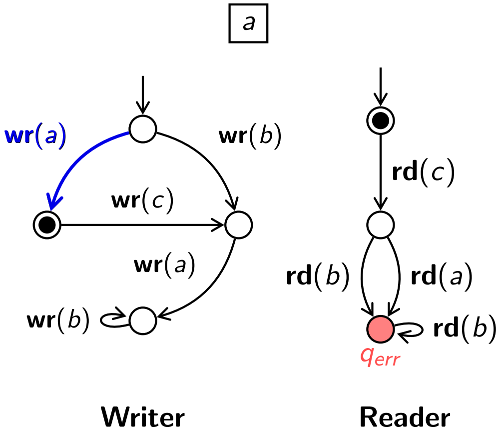

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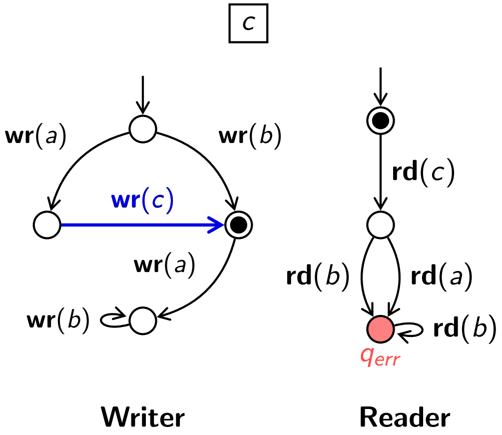

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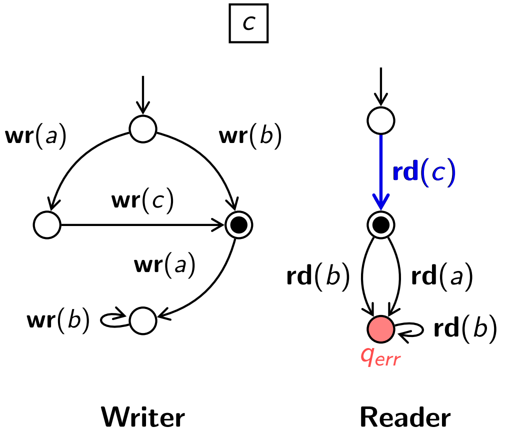

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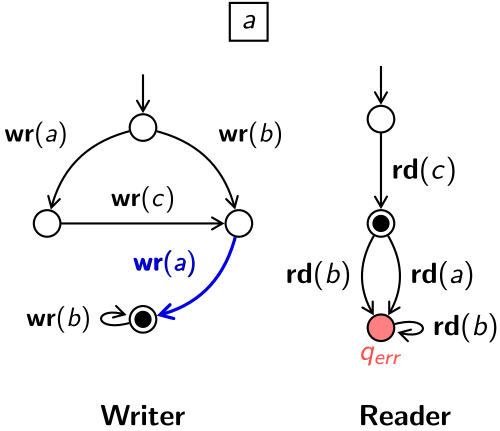

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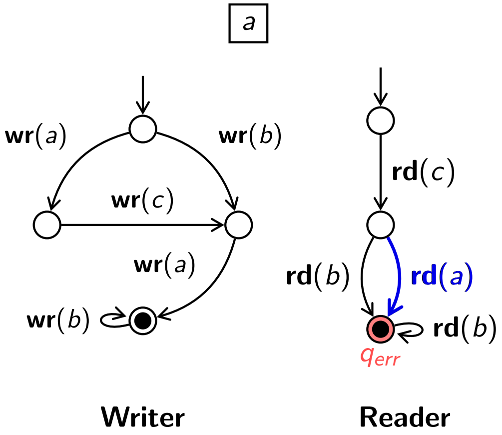

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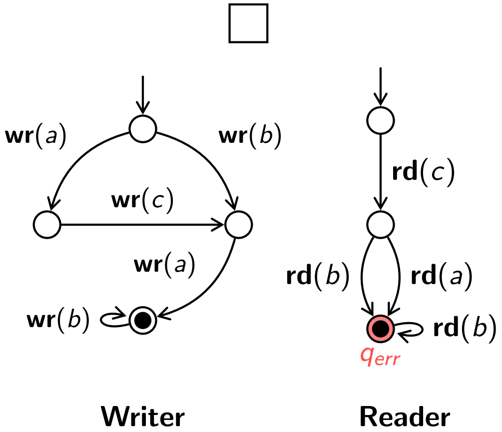
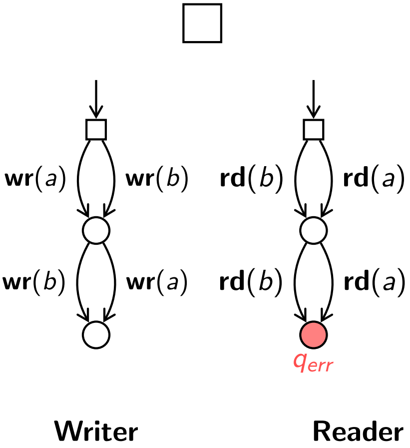


Local runs of writer and reader can be combined whenever the sequence read by Reader is a **subword** *of the sequence written by Writer.*

Ex: *bab* is a subword of $a\underline{b}b\underline{ab}a$.

Distributed synthesis

- Each process has Controller and Environment states.



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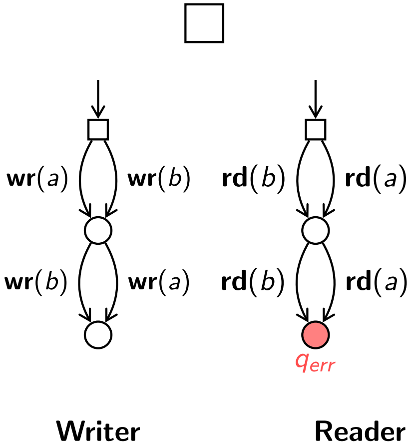

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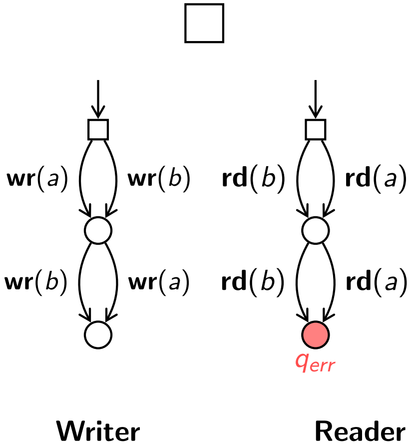
- Local strategy = function $\sigma_w : \mathit{Paths}_w \to \mathit{Transitions}_w$ that chooses the next transition from controllable states based on past transitions.



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there exists an **invariant** $\mathcal{I} \subseteq \Sigma^*$ *such that*

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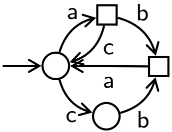
- $\mathcal{I}$ a priori arbitrarily large
- If we were given $\mathcal{I}$ $\rightarrow$ reduces to *regular two-player safety game*

Objective-independent memory

Arena $=$ 

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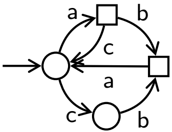
Arena $=$ 

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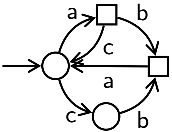

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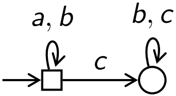

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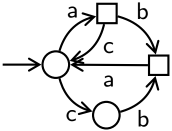
Arena $=$ 

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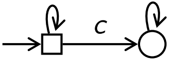

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$\mathcal{C} \subseteq 2^{\Sigma^*}$ has *objective-independent memory* if $\exists f : \mathbb{N} \to \mathbb{N}$, $\forall$ arena $\mathcal{G}$ and $W \in \mathcal{C}$, if Controller has a winning strategy for W in $\mathcal{G}$, then she has one with memory $\leq f(|\mathcal{G}|)$.

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"Easy" extension: everyone can read and write, but the process writing can change $\leq K$ times.

Theorem

The distributed synthesis problem is NEXPTIME-complete for shared-memory systems with bounded writer switches.

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Thank you for your attention!