Responsibility in verification

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Joint work with Christel Baier, Florian Funke, Simon Jantsch, Stefan Kiefer, Karoliina Lehtinen

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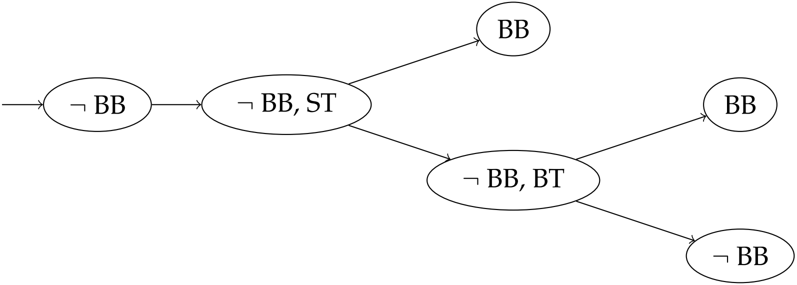
How do we distribute responsibility fairly?

Previous work

Their recurring example involves two characters, Suzy and Billy, throwing stones at a glass bottle. We use three variables, BB (the bottle is broken), ST (Suzy throws a stone) and BT (Billy throws a stone).



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The degree of responsibility of state s with respect to a is $\frac{1}{m}$, where m is the minimal size of a set of states containing s such that flipping the value of a in those states makes the system fail the specification.

Voting

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Responsibility is often understood either in a binary way (responsible or not), or in a weighted way (we distribute responsibility).

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We focus on games such that $\nu(T, P)$ only depends on T.

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The **Shapley value** is defined as:

$$\mathrm{Sh}(a) = \frac{1}{n!} \sum_{\pi \in \Pi_n} \nu(\pi_{\geq a}) - \nu(\pi_{\geq a} \setminus \{a\})$$

Shapley values

The Shapley value function is the only one satisfying the following conditions.

1. **Efficiency** $\sum_{i=0}^{N} Sh_i(\nu) = \nu(\{1,\ldots,N\},\{\{1,\ldots,N\}\})$
2. **Symmetry** Renaming players does not affect their rewards.
3. **Additivity** For all games μ, ν, and $C \in \mathbb{R}$, $Sh_i(C\mu + \nu) = CSh_i(\mu) + Sh_i(\nu)$, i.e., Sh_i is a linear function.
4. **Null-Player Axiom** If for all $(T,P) \in \mathcal{C}(N)$, $\nu(T \cup \{i\}, P_{T\leftarrow i}) = \nu(T,P)$ then $Sh_i(\nu) = 0$.

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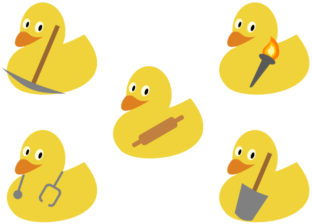
- $\rightarrow$ Agents stop cooperating one by one in a random order.



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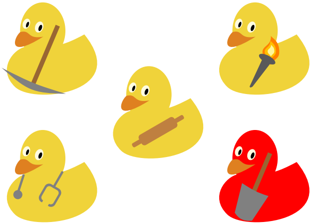

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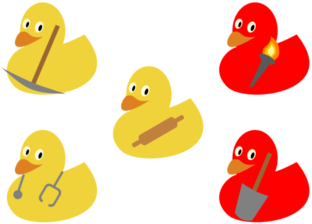

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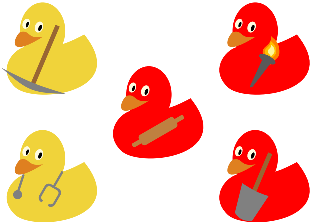

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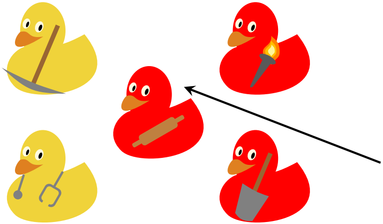

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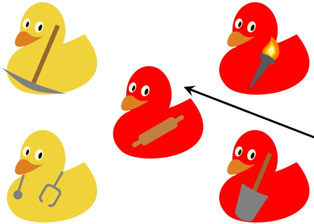

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The importance of an agent is its probability to be decisive.

LTL model checking and importance of states

LTL is a logics designed to express properties of infinite words.

$$\varphi ::= a \mid \varphi \wedge \varphi \mid \neg\varphi \mid X\varphi \mid \varphi U \varphi \mid G\varphi \mid F\varphi$$

For instance, GFa expresses that at all positions of the word there is a further position at which a is true.

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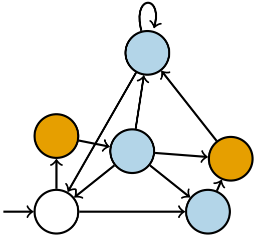

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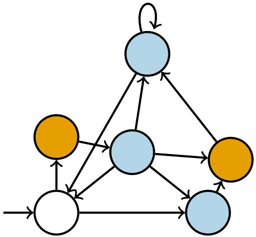

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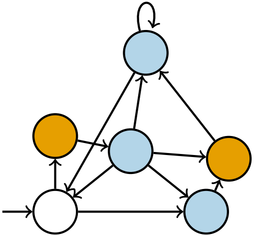

Which states of K are important for φ?

A state of K is (more) important for φ if **its nondeterminism matters (more)**.

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Formalizing importance for LTL

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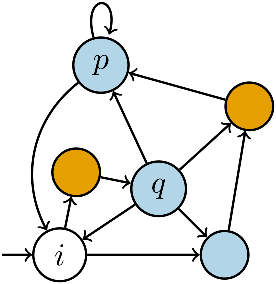

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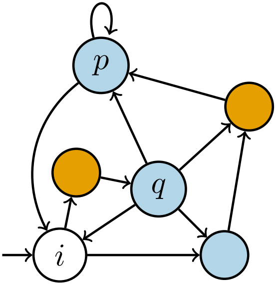

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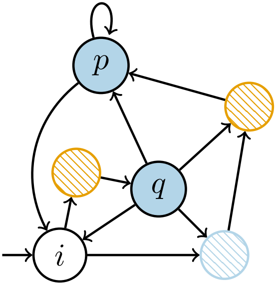

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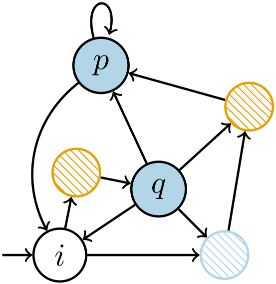

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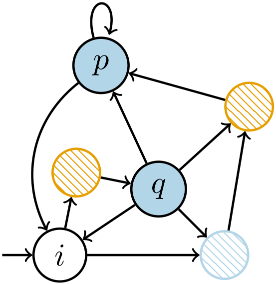

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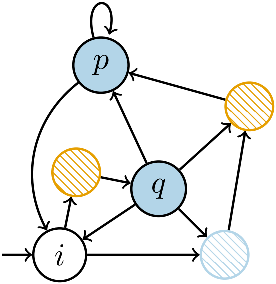

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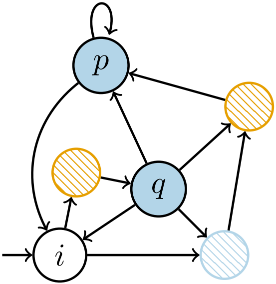

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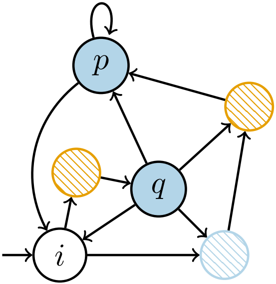

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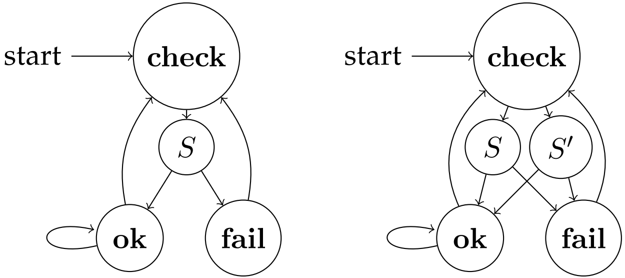
$$\mathcal{I}(i) = \mathcal{I}(q) = 1/6$$

Comparison

A server is tested by sending requests. If the server fails to answer, it is tested again, otherwise the system may wait before testing again.



$\varphi = GF\,\textbf{check} \wedge FG\,\neg\textbf{fail}$

We obtain an importance of $\frac{1}{2}$ for S and ok in the first example, $\frac{1}{2}$ for ok and $\frac{1}{6}$ for $check$, S and S' in the other.

Complexity of determining importance

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Is C enough to make the system work?

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Importance computation problem. Given $q \in S$:

compute $|S|! \cdot \mathcal{I}(q)$

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TABLE I
A SUMMARY OF THE RESULTS ON THE COMPLEXITY OF THE VALUE, USEFULNESS AND IMPORTANCE PROBLEMS FOR VARIOUS TYPES OF SPECIFICATIONS.

	Büchi	**Rabin**	**Streett**	**Parity**	**Explicit Muller**
Value	P	NP	coNP	$\in$ NP $\cap$ coNP	P
Usefulness	NP	Σ_2^P	Σ_2^P	NP	NP
Importance	#P	$\#P^{NP}$	$\#P^{NP}$	#P	#P

	Emerson-Lei	**LTL**	**2-turn CTL**	**Concurrent CTL**
Value	PSpace	2ExpTime	Σ_2^P	$\in$ ExpTime
Usefulness	PSpace	2ExpTime	Σ_3^P	$\in$ ExpTime
Importance	PSpace	2ExpTime	$\#P^{\Sigma_2^P}$	$\in$ ExpTime

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- Probabilistic approximations are enough for our purpose: Just draw orders at random and do a dichotomic search for the critical state.

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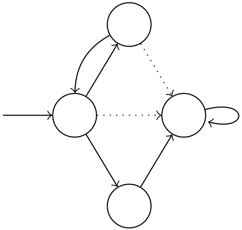

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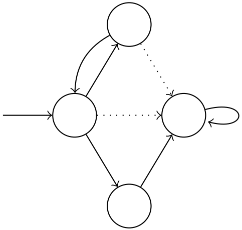

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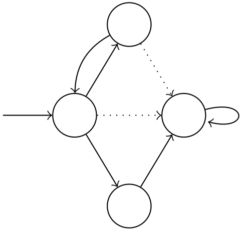

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We considered **one-shot** games instead.

CTL complexity

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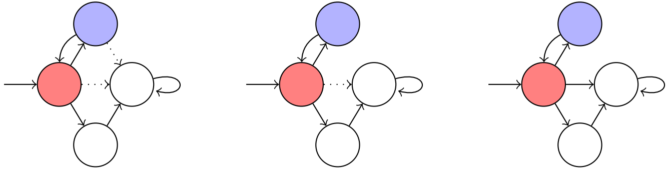
1) One player chooses all its transitions, then the other one does.



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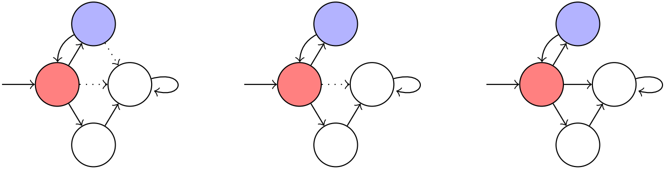

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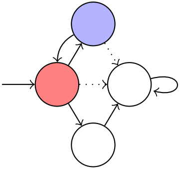

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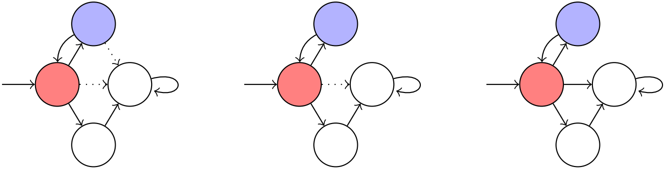

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- Computing the value of a set of states comes down to solving a linear optimization problem with exponential input.

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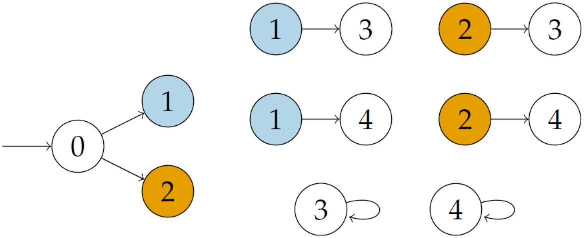
Definition

A *linear strategy* is a function $\sigma : E^* \to E$.

Sat (resp. Unsat) wins if they have a strategy to guarantee that the resulting tree (un)satisfies the objective. The game is sometimes undetermined.

Example of undeterminacy



In this example we consider the CTL formula $EF3 \wedge EF4$.

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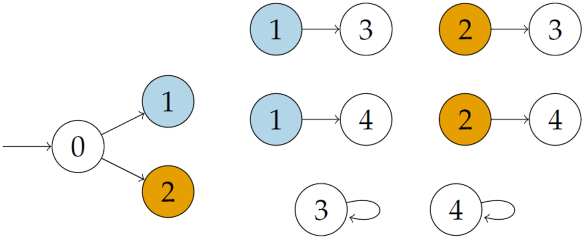

In this example we consider the CTL formula $EF3 \wedge EF4$.

Question

Which tree objectives allow tree games to be determined?

Other examples

$\rightarrow EFa$ is a determined language.

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$\rightarrow$ $AG(a \vee EFb)$ as well.

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$\rightarrow$ $AG(a \vee EFb)$ as well.

$\rightarrow$ $AG(EFa \wedge EFb)$ is not.

Game automata

Definition

A *game automaton* is an alternating tree automaton in which a pair (q, i) appears at most once in each transition.

Proposition

Languages expressed by game automata yield determined tree games.

However there exist languages non expressible by game automata which yield determined tree games.
It is the case with the language of trees having countably many branches fully labelled with a.

Continuation

$\rightarrow$ Fragment of CTL for which tree games are determined.

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- → Probabilistic approximation methods for Shapley values in this framework.

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- $\rightarrow$ Extension to probabilistic systems.

Continuation

- → Fragment of CTL for which tree games are determined.
- → Probabilistic approximation methods for Shapley values in this framework.
- → Extension to probabilistic systems.
- → Control point of view (make the adversary all-knowing)

Thank you for your attention!